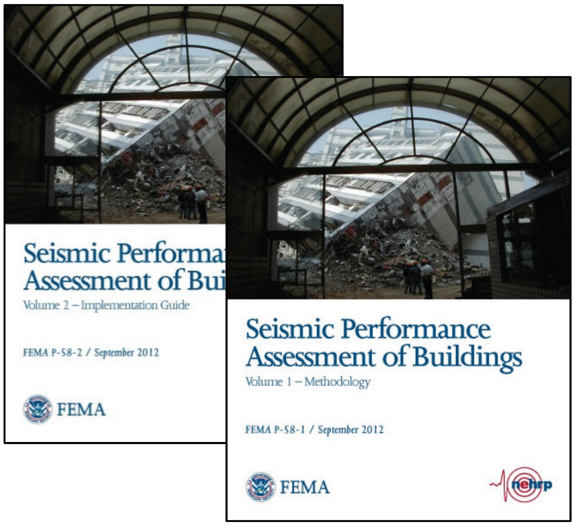

Background Document

FEMA P-58/BD-3.7.11

Risk Management Products Team Report: PACT Verification and Sensitivity Studies

Prepared by

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U.S. Department of Homeland Security
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October 6, 2008

FEMA

nehrp

Background Documentation

FEMA P-58 Background Documents are a series of reports documenting the technical background and source information for key aspects of the FEMA P-58 methodology and its implementation. These reports were developed over the course of the 10-year ATC-58/ATC-58-1 Projects funded under FEMA Contracts EMW-2001-RP-0056 and HSFEHQ-06-D-1105.

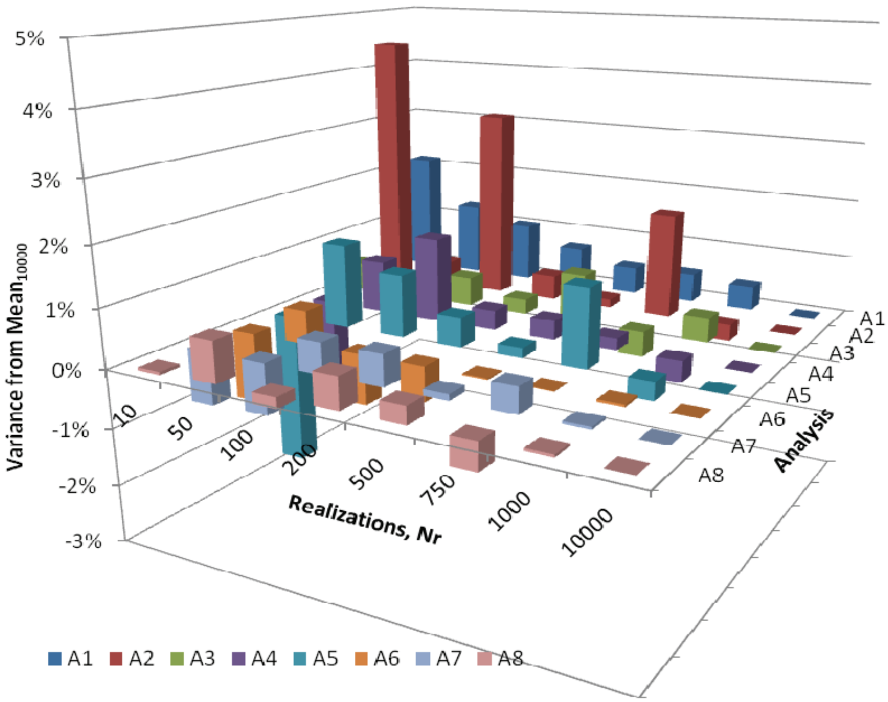
Background Documents were developed by consultants, serving at various levels within the project hierarchy, reporting the results of: (1) decisions on technical development protocols; (2) focused studies on the development of key aspects of the methodology; (3) documentation of recommended procedures; and (4) collection of available data for the development of structural and nonstructural fragilities. They were initially intended to serve as a record of the technical state-of-knowledge at the time they were produced, and as resources for the development of the eventual project reports. As such, they represent a snapshot in time, and may, or may not, match the technical content, recommended procedures, or data incorporated into the final methodology and its implementation.

This Background Document is intended for the purpose of providing supplemental knowledge to users of the FEMA P-58 methodology. Information contained herein has not been independently verified for accuracy as a stand-alone document, and may have been superseded in its final implementation within the methodology. Users of information in this document assume all liability arising from such use.

Notice

Cover illustration – Primary resource documents for the FEMA P-58 *Seismic Performance Assessment of Buildings, Methodology and Implementation* series of products: FEMA P-58-1, *Volume 1 – Methodology*, and FEMA P-58-2, *Volume 2 – Implementation Guide*.

Risk Management Products Team Report: PACT Verification and Sensitivity Studies



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Risk Management Products Team Report: PACT Verification and Sensitivity Studies

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WJE | ENGINEERS ARCHITECTS MATERIALS SCIENTISTS

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PACT Verification and Sensitivity Studies

INTRODUCTION

This report describes a number of studies and analyses performed under the Risk Management Products (RMP) group of ATC-58. These studies focused on verification and sensitivity analysis using the computer program PACT. Issues considered include the correctness of the computations performed by PACT, an investigation of the number of realizations and intensities required for accurate and stable loss calculations, and a study investigating the contribution of uncertainties in demand, capacity and cost to the final loss calculation results. The latest available version of the PACT software (PACT Beta 1.00 with patch 2 dated 4 September 2007) was used in most studies, while a modified version of the software that is capable of exporting internal data was used to perform verification of loss calculations.

VERIFICATION OF PACT CALCULATIONS

In order to verify the internal calculations performed by PACT, a modified version of PACT was provided by John Martin and Associates, the developers of PACT, that supports the capability to export internal data and generated random numbers that are used in the loss calculations. The results exported by PACT are intermediate results generated during the loss calculations. The exported data was examined under different analysis situations in order to verify the correctness of the calculations, and confirm their consistence with the ATC-58 document.

The calculations of losses by PACT involves a number of internal calculations and steps, which include the generation of random numbers, the computation of engineering demand parameters (EDPs) for each realization, the computation of damage states for each performance group, and performing the final realization or time-based loss calculation. The overall calculation procedure is described in the ATC-58 document. In order to verify the correctness of the calculations performed by PACT and to confirm their consistency with the published documentation, a number of calculations and verifications were performed.

Referring to section F.2.2 of the ATC-58 document (35% draft), the following equations can be written (using Matlab syntax).

Y = log(X), where X is a matrix of analysis demand values for a given intensity of size (m x n)
M_Y = mean(Y), mean values (over m analyses) of the n EDP's
D_Y = diag(std(Y)), the diagonal matrix containing the standard deviations of the n EDP's
R_{YY} = corrcoef(Y), the correlation coefficient matrix
L_Y = chol(R_{YY}), the Cholesky decomposition of R_{YY}.

Given a vector of random normal variables U with a mean of 0 and standard deviation of 1, new realizations can be generated using:
$Z = D_Y L_Y U + M_Y$,
W = exp(Z), where W contains the generated EDP's for this realization.

Currently, the exported results from PACT consist of the following:

1. Matrix of normal values U (m x $N_{realizations}$) used for computing EDP values. A different matrix of random normal numbers is generated for each run and direction.

2. Matrix of EDP values (m x $N_{realizations}$). A different EDP matrix is generated for each run and direction.
3. Fragility raw damage random variables. The random raw damage percentages, for each performance group (PG) in each run and direction.
4. Fragility raw damage states. The final damage states, for each PG in each run and direction.

The calculation of the above values was verified by checking selected runs, and independently computing and comparing the above quantities to those calculated by the program.

Generation of Engineering Demand Parameters (EDP's)

EDP's for each realization can be obtained using the equations above. The computation of the EDP values from the random values (U) was verified and compared to independent calculations performed in Matlab using the same random values. A check was performed for the PACT office building example, using one of the runs (direction 1 of Run 4) with the default number of 200 realizations. The PACT EDP's were compared to the independently computed EDP's, and the average difference was found to be about 0.04%, while the maximum error was 0.47%, which is most likely due to round-off error, or to slight differences in the algorithms for the computation of the correlation matrix and Cholesky decomposition between PACT and Matlab. The generated EDP's were also compared to the original demands as shown in the following tables. Note the all of the ratios are very close to 1.0 indicating that the generated realizations have a similar distribution to the original EDP's.

Table 1: Ratio of simulated to original logarithmic means

δ_1 (%)	δ_2 (%)	δ_3 (%)	a_1(g)	a_2(g)	a_3(g)	a_4(g)
1.0073	1.0052	1.004	0.9646	0.9372	0.9649	1.026

Table 2: Ratio of entries in simulated and original R_{YY} matrices

1.000	0.995	1.000	1.349	1.031	1.080	0.927
0.995	**1.000**	1.004	0.661	1.042	1.056	0.955
1.000	1.004	**1.000**	0.861	1.126	1.116	0.963
1.349	0.661	0.861	**1.000**	0.993	0.915	1.046
1.031	1.042	1.126	0.993	**1.000**	1.018	1.007
1.080	1.056	1.116	0.915	1.018	**1.000**	1.097
0.927	0.955	0.963	1.046	1.007	1.097	**1.000**

Computation of Damage States

The damage state for each PG in a given realization and direction can be calculated from the EDP and the fragility curve for the PG. A fragility random variable is used to select one of the possible fragility damage states, based on the relative likelihood of each state. The fragility random variables and damage states exported by PACT were examined (in conjunction with the corresponding EDP). When the damage states are correlated, this reduces to a single random variable and a single damage state per performance group for each run and direction at a given floor, and hence is relatively easy to verify. When the damage states are not correlated, the damage is computed as the percentages of the elements in each damage state. This requires a large amount of data to be exported in order to perform the verification.

The computation of damage states was checked for the first case (correlated), but not for the uncorrelated case since the current PACT version does not export all the required data. For the correlated damage case, the reported damage states were found to be consistent with the provided random numbers. This issue should be revisited in order to confirm that the damage states are being computed correctly, particularly for uncorrelated damage. It should be noted that the relevant PACT calculations will be altered and expanded during the next phase of PACT development, which will include treatment for more complex fragility relationships including simultaneous and mutually exclusive damage states. A framework for verifying the correctness of these computations is recommended.

Cost of Repair Calculations

Detailed verification of loss calculation could not be performed because the required data is not currently exported from PACT. However, some simple global checks were performed using simplified input, and the results were found to be correct. This was done by using a deterministic realization input (by setting all random variables for computing EDP's to zero), and also setting the fragility beta to a very small number, which resulted in all elements on any given floor to be in the same damage state in every realization, making it possible to compute a total cost of repair by hand. An export feature would be needed in future versions of PACT in order to perform detailed verification calculation of repair costs.

Discussion

The above calculations provide a reasonable degree of confidence in the calculations performed by the current version of PACT. While some of the calculations suggest that some of the computations need to be checked, it appears that the calculations performed by PACT are generally correct. The calculations were checked up to the stage of damage state calculation, and the checking of subsequent calculations involving cost of repair, including random cost variation, was not thoroughly checked, although some spot checks were performed which correspond to simple cases, and these checks did not reveal any errors. In order to gain a greater degree of confidence in the loss calculations provided by PACT, some of the limitations of this study should be addressed in the future. As suggested above, it is recommended that a framework be developed for checking PACT calculations, particularly those relating to determination of damage states. Additionally, there are other issues that still need to be resolved which the development team is aware of. For example, the cost computation currently in PACT does not consider damage in more than one direction when counting the repair quantity for determining unit cost.

SENSITIVITY OF COMPUTED LOSS TO DIFFERENT TYPES OF UNCERTAINTY

Two sensitivity analysis studies were performed to investigate the relative contributions of different sources of uncertainty to overall variability of loss calculations. Knowing the more significant contributors to uncertainty makes it possible to improve estimates by focusing on those variables that have the largest impact. One of the two studies considered uncertainty associated with cost and with fragility damage thresholds while keeping the demand fully stochastic, while the second study also considered the effect of uncertainty in the demand estimates.

Sensitivity to Uncertainty in Cost and Capacity

Uncertainty in cost and capacity was controlled by modifying the dispersion and beta factors for cost functions and fragility damage states, respectively. The analysis was performed by setting all beta and dispersion values to zero and running the analysis to obtain a median estimate. Then, the mean cost or fragility variable was set to the 10 percentile value, one variable at a time, and the analysis performed. The same analysis was repeated with the 90 percentile values. Since the model still had variability due to seismic demand parameters being different in each realization, a large number of realizations was used (1000 to 10000), and the mean of several runs, typically 5, was obtained and reported. Additional cases were conducted in which all cost related variables were concurrently set at their 10 or 90 percentile values at the same time, in addition to cases where all fragility related variables were set at their 10 or 90 percentile values.

Figure 1 shows a tornado diagram comparing the sensitivity of the annualized cost to uncertainty due to different factors. The factors are sorted in decreasing order of influence. Cases where all of the capacities or all of the costs were varied concurrently show the largest influences as expected, followed by other factors such as ceiling system damage state uncertainty, partition damage state uncertainty, partition repair cost uncertainty, and SMF damage state uncertainty.

Figure 2 and Figure 3 compare the influence of all factors related to damage uncertainty and cost uncertainty, respectively. In general, it appears that for annualized cost, the uncertainty in partition and ceiling cost and capacities have a greater contribution to total uncertainty than other components, directly followed by uncertainty in the SMF capacity. It may also be useful to consider the results of this study for scenario or intensity-based loss calculations, since other variables may become more important in those cases.

While these results are obviously specific to the problem analyzed, which in this case is the default three-story office building, the methodology and type of analysis can be extended to any problem. Such functionality can be incorporated into PACT which would automate the generation of sensitivity analysis results such as the tornado diagrams in Figure 1 through Figure 3.

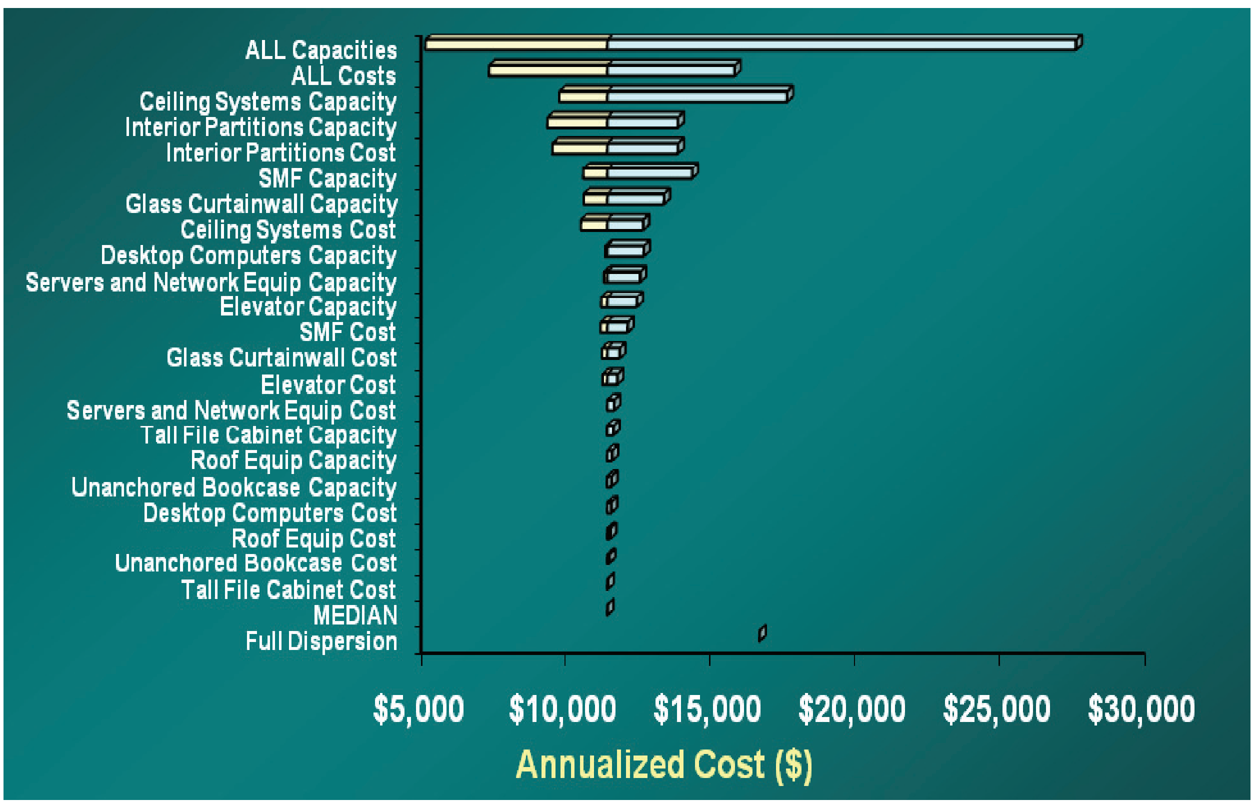

Figure 1. Sensitivity of annualized cost ($) to damage state and repair cost uncertainty.

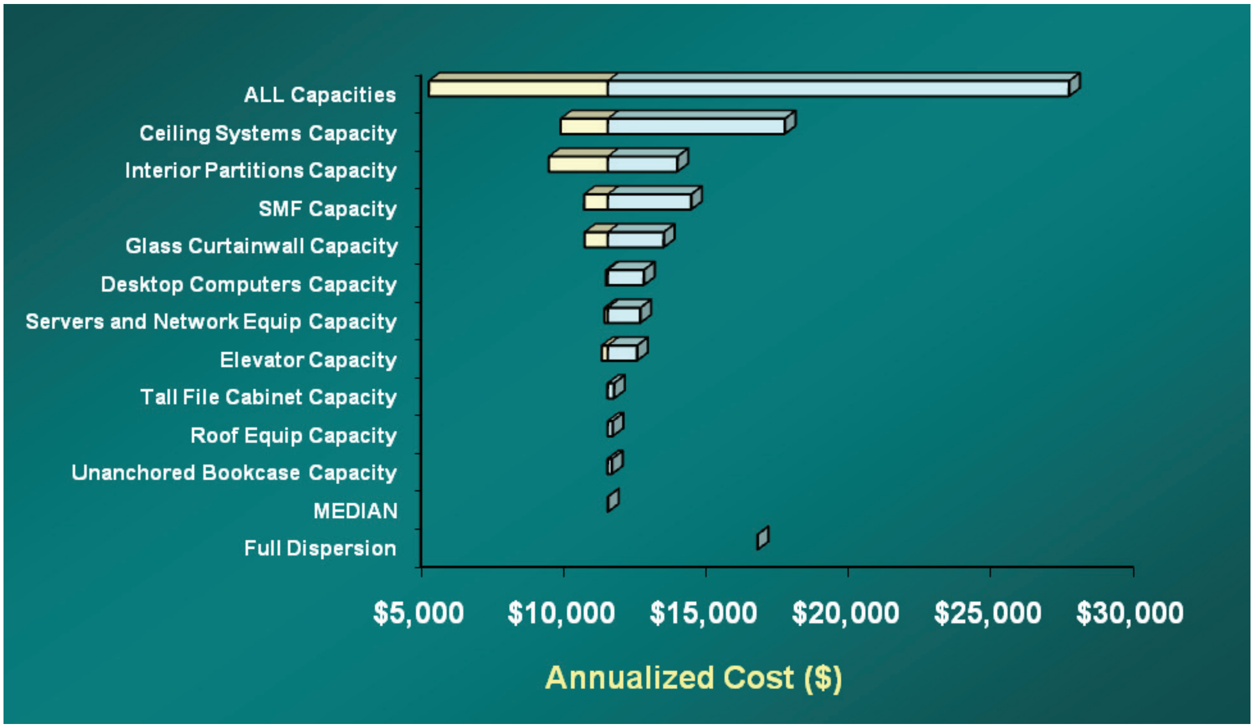

Figure 2. Sensitivity of annualized cost ($) to damage state uncertainty.

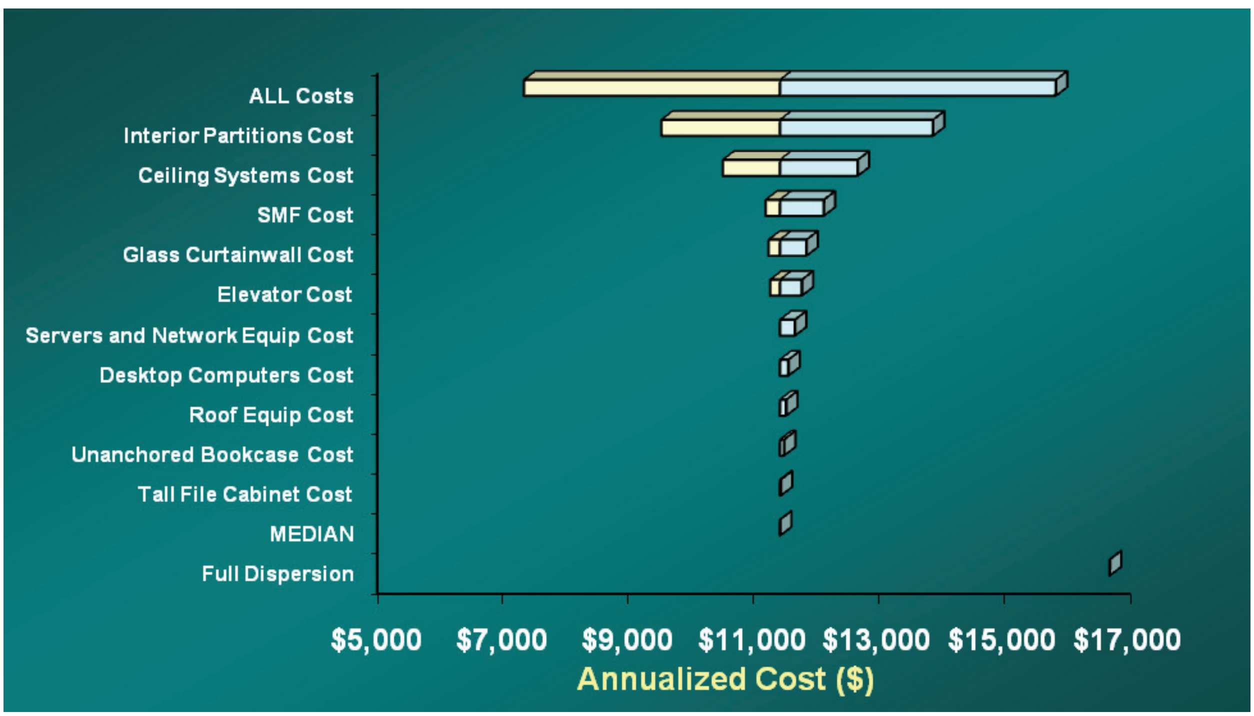

Figure 3. Sensitivity of annualized cost ($) to repair cost uncertainty.

Sensitivity to Uncertainty in Cost, Capacity and Seismic Demand

A second parametric study was performed to measure the sensitivity of the computed time-based loss to variation in the assumed median values for capacity, cost and seismic demand parameters. In each case, the dispersion was set to zero for capacity, cost and EDP calculations, and the median values for damage state, cost of repair and realization EDP were set at either 10% or 90%. The variables were generally set one at a time to the 10% or 90% value, with all remaining variables set at the 50% (median) value. Hence, most of the runs are essentially deterministic, which means that only a single realization is required. Some runs were performed with the default dispersions for one or more of the main variables in order to provide a point of comparison. For example, in Figure 4, the case "50% Cap, Cost & Demand" refers to a fully deterministic analysis with median values (and zero dispersion) for all capacities, costs and demands. Case "50% Cap & Cost/Stoch Demands" is a case in which median values and zero dispersion are used for all capacities and costs, but demands are fully stochastic, hence a large number of realizations were used in order to get a stable result.

Figure 4 through Figure 6 show results that are equivalent to those in Figure 1 through Figure 3. The current results however are generally deterministic because the demand values also have zero dispersion in this case. Note that in generally, reducing the dispersion results in a smaller final annualized cost, as compared to the previous study. Also, as dispersion is used for one or more of the variables (capacity, cost, demand), the resulting annualized cost becomes higher.

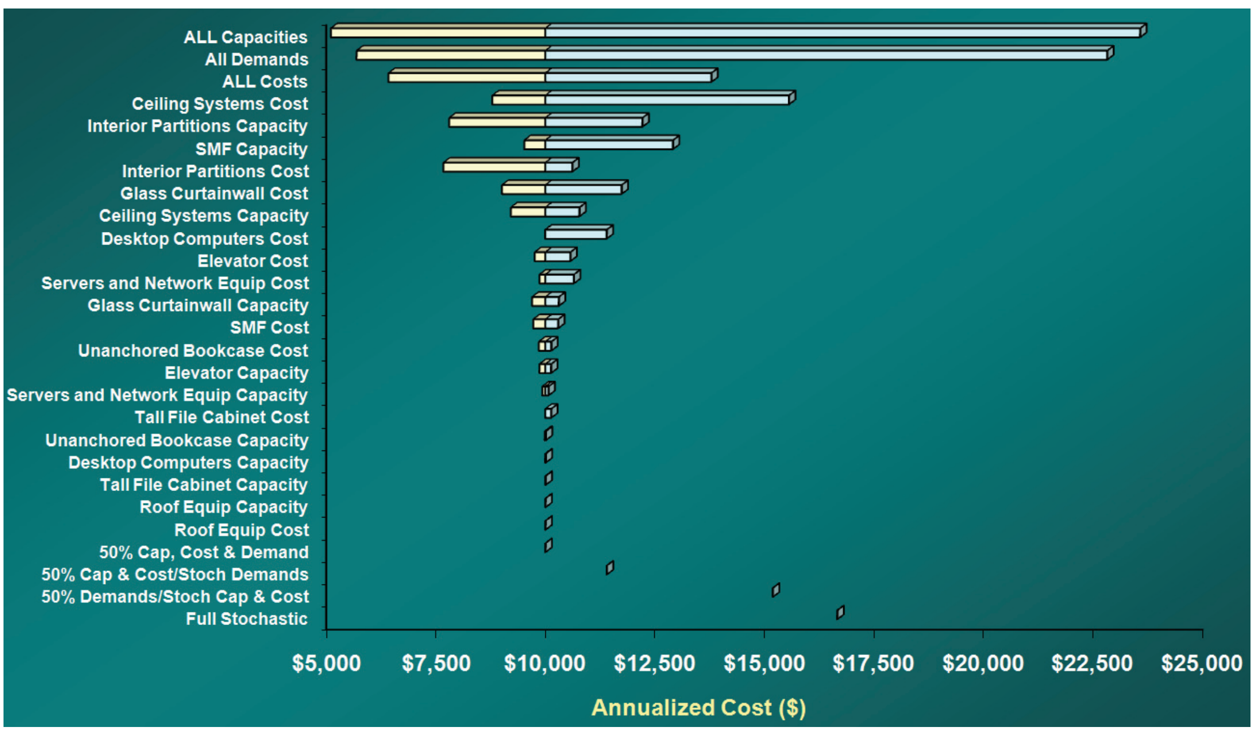

Figure 4. Sensitivity of annualized cost ($) to damage state, repair cost, and demand uncertainties.

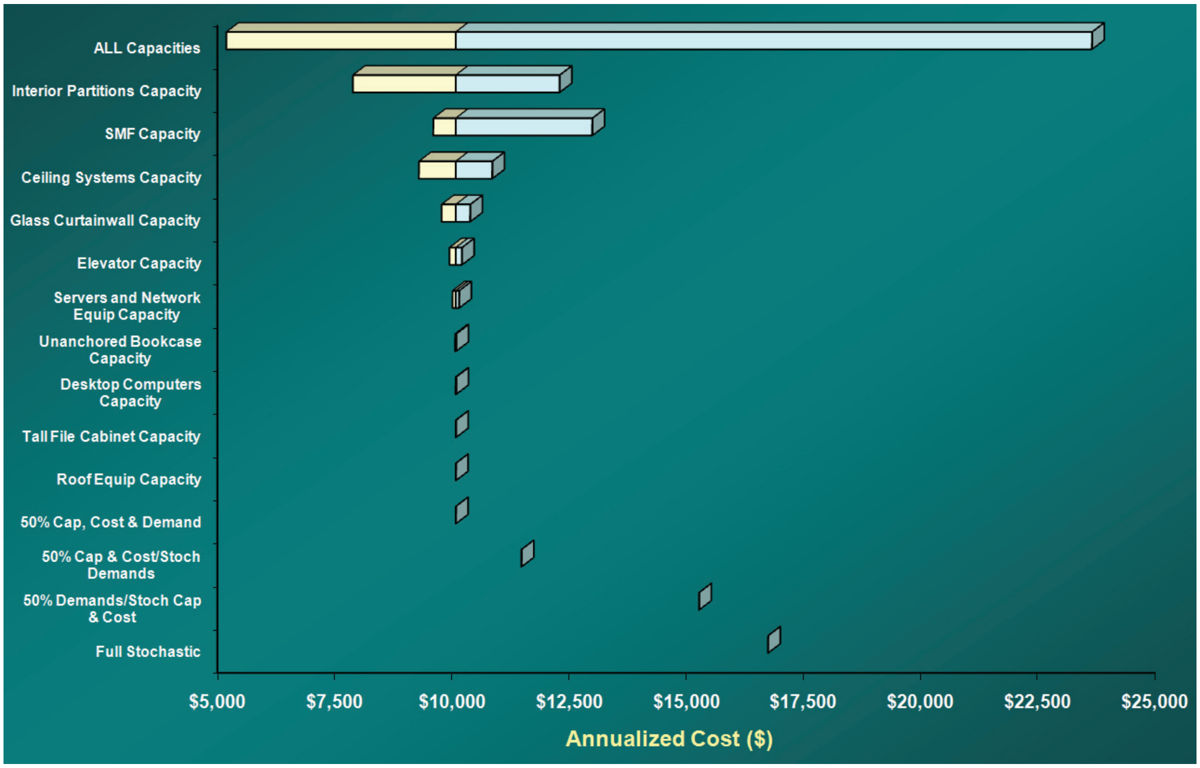

Figure 5. Sensitivity of annualized cost ($) to damage state uncertainty.

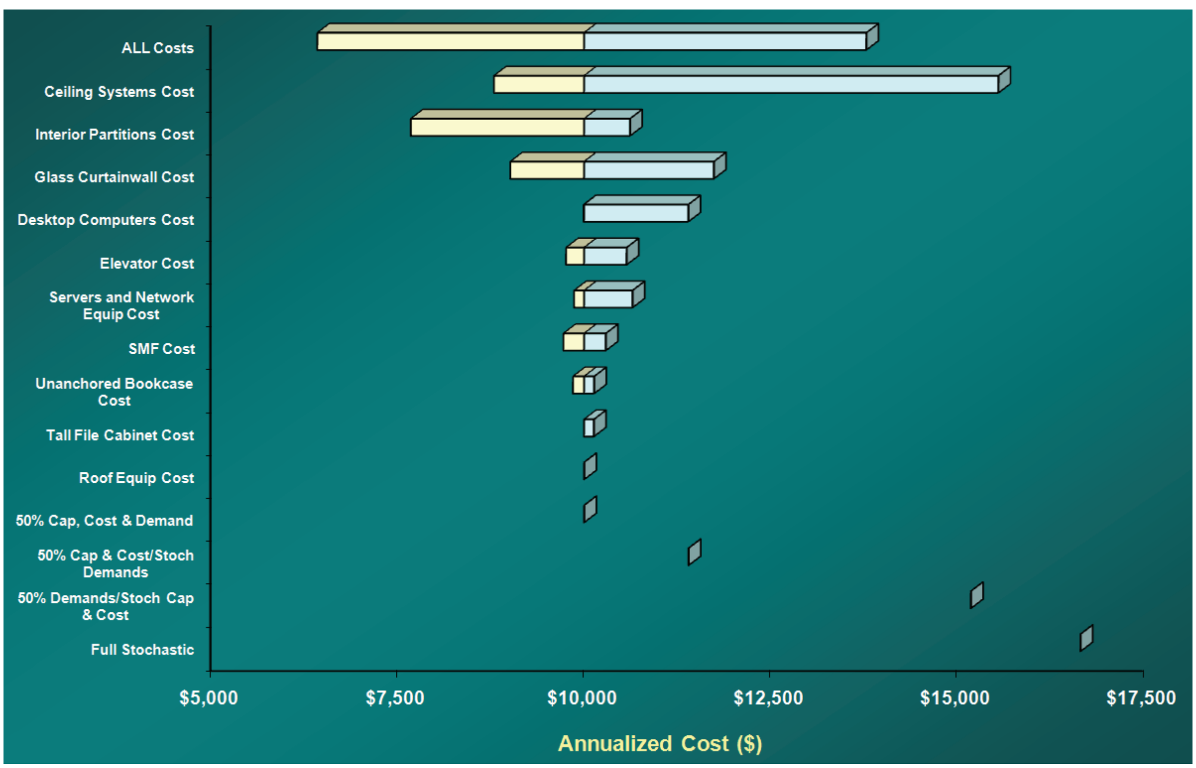

Figure 6. Sensitivity of annualized cost ($) to repair cost uncertainty.

NUMBER AND SELECTION OF INTENSITIES FOR TIME BASED ANALYSIS

This parametric study considered the number and distribution of intensities used for time-based loss analysis. This issue is important because it directly relates the total number of separate nonlinear history analyses that are needed. The PACT office building example included 8 different intensities. Four different scenarios were considered and compared to the original scenario. The different scenarios included the following cases:

- Time-based analysis using intensities 2, 4, 6, and 8.
- Time-based analysis using intensities 1, 3, 5, and 7.
- Time-based analysis using intensities 2, 5, and 7.
- Time-based analysis using intensities 3, and 6.

Time based analyses were performed using each of the above intensities in addition to the original example. Results were compared both on an annualized basis as well as for the shape and magnitude of the loss curve. The resulting hazard curve with each scenario is illustrated for the four scenarios in Figure 7 to Figure 10. It is apparent that removing intensity values in regions of the hazard curve that exhibit large slope variations can significantly affect the loss calculation results. The resulting loss curves given the various scenarios are shown in Figure 11 and Figure 12. It is clear that the selection of intensities has a significant impact on the shape of the loss curve. As a result, the annualized loss is also affected as shown in Figure 13, although the magnitude of the change is dependent on the intensities selected since the shape of the loss curve can change significantly without significant change in the area under the curve if some regions of the curve go up while others go down. In general, while it seems that reducing the number of intensities can have a significant effect, the values and distributions of these intensities is a more important factor. This should be the subject of further investigation.

In summary, the results clearly show a significant effect of the number and distribution of intensities on the results of the time-based loss estimate. The intensity selection affects both the shape and values of the loss curve, and in general, it appears that reducing the number intensities should be avoided when possible. It is recommended to investigate this issue further to determine whether more efficient integration algorithms or intensity selection are attainable. In order for this task to be performed, more nonlinear analysis results at different intensities will be required.

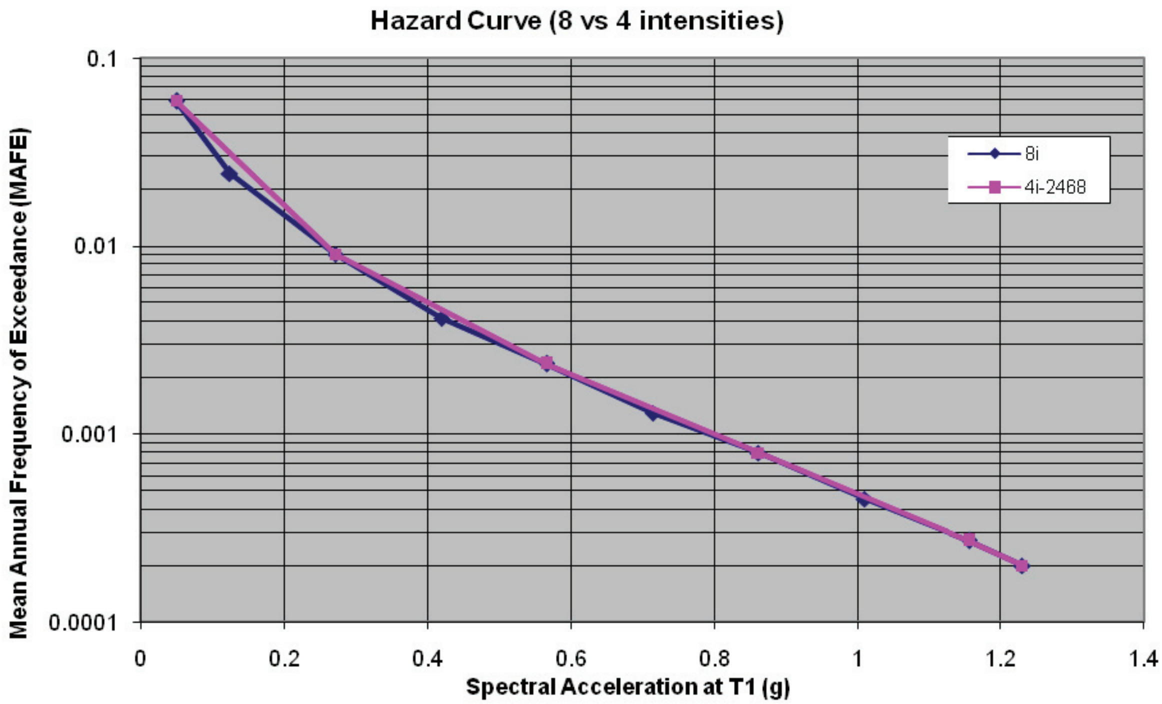

Figure 7. Comparison of hazard curve for 4 intensities (2, 4, 6, & 8) vs. 8 intensities.

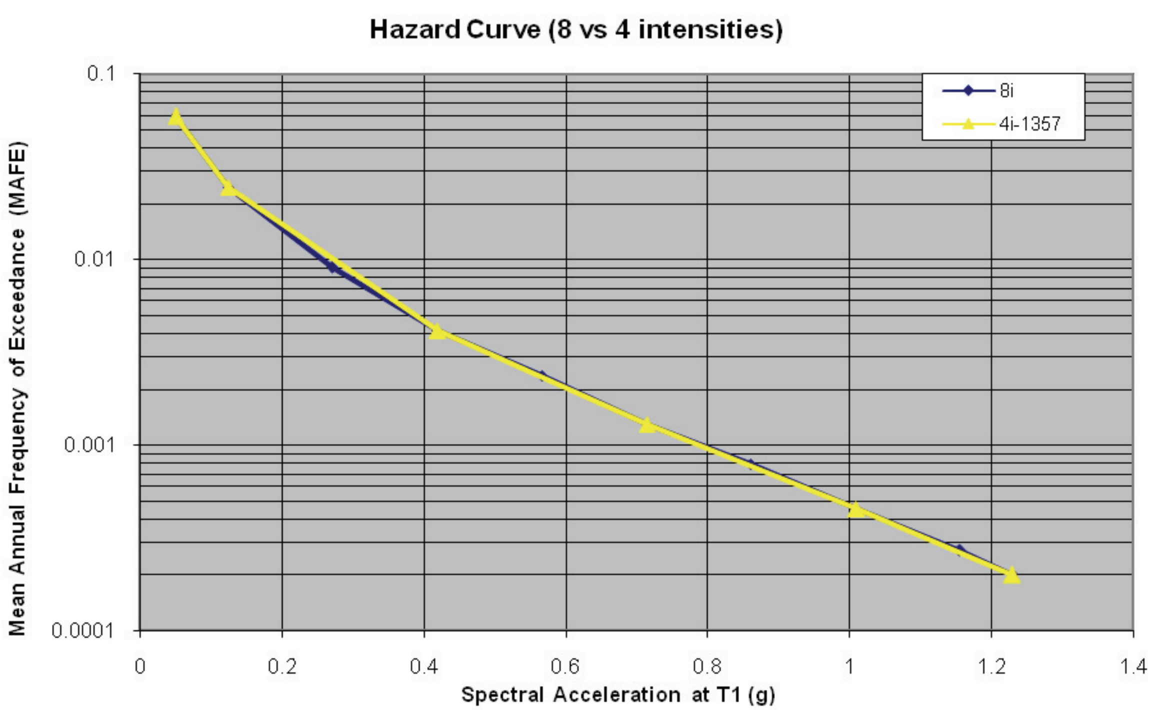

Figure 8. Comparison of hazard curve for 4 intensities (1, 3, 5 & 7) vs. 8 intensities.

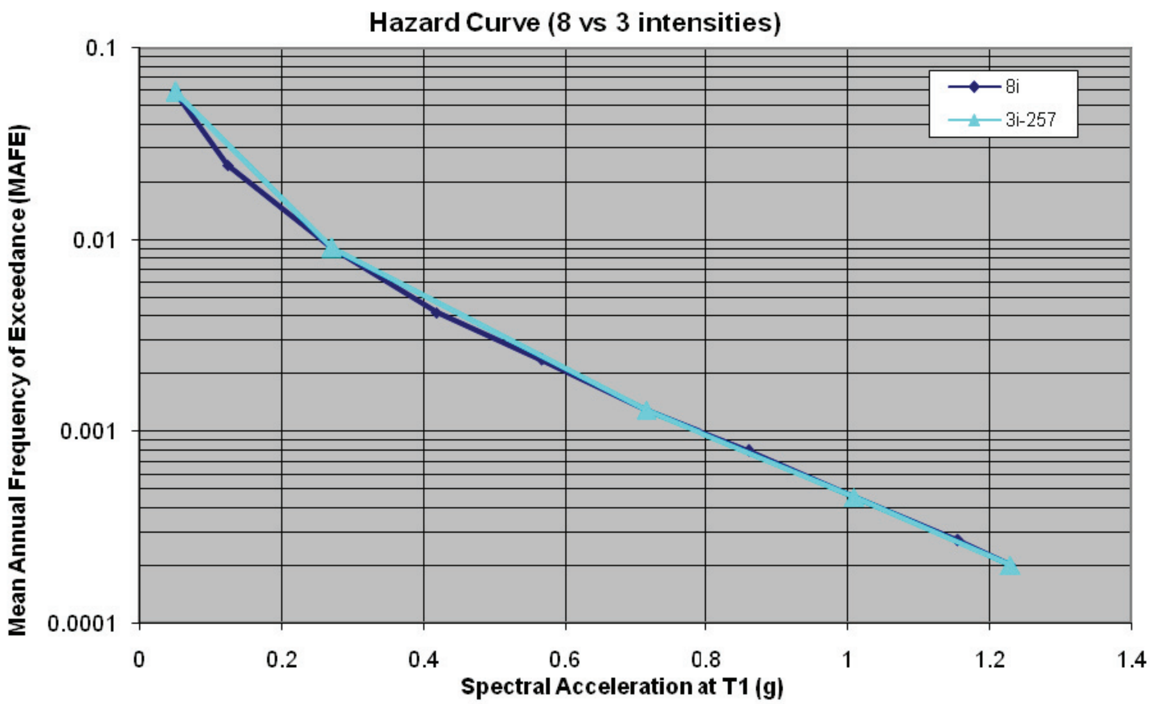

Figure 9. Comparison of hazard curve for 3 intensities (2, 5, & 7) vs. 8 intensities.

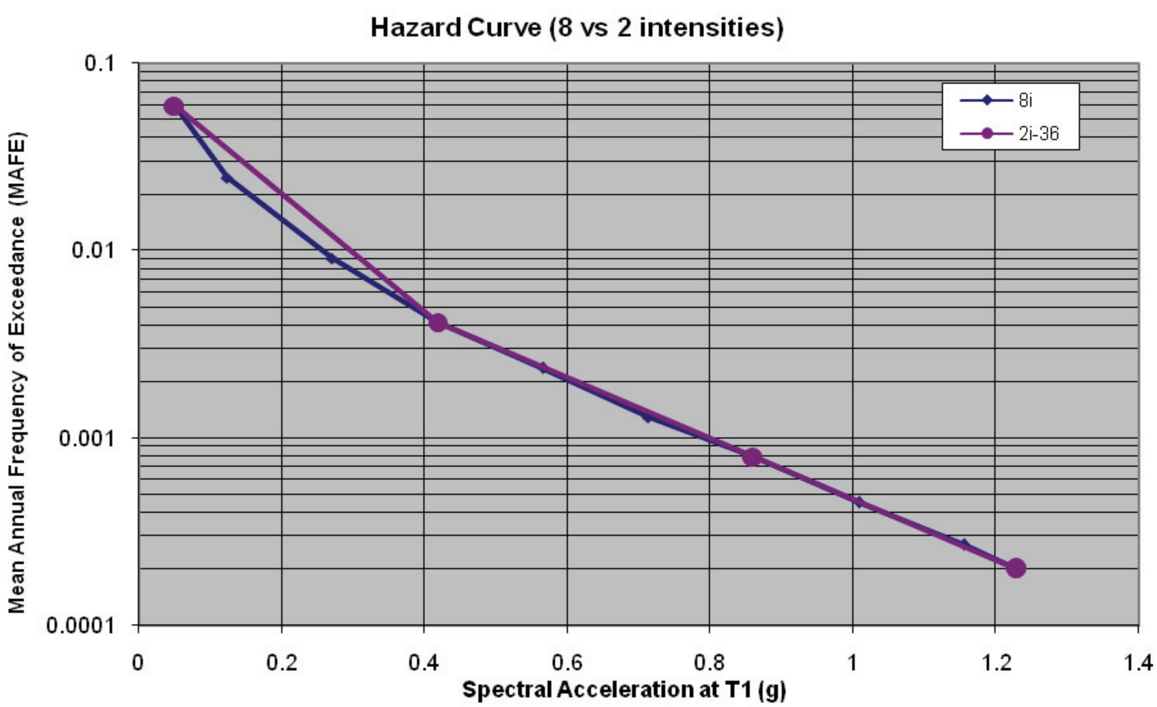

Figure 10. Comparison of hazard curve for 2 intensities (3 & 6) vs. 8 intensities.

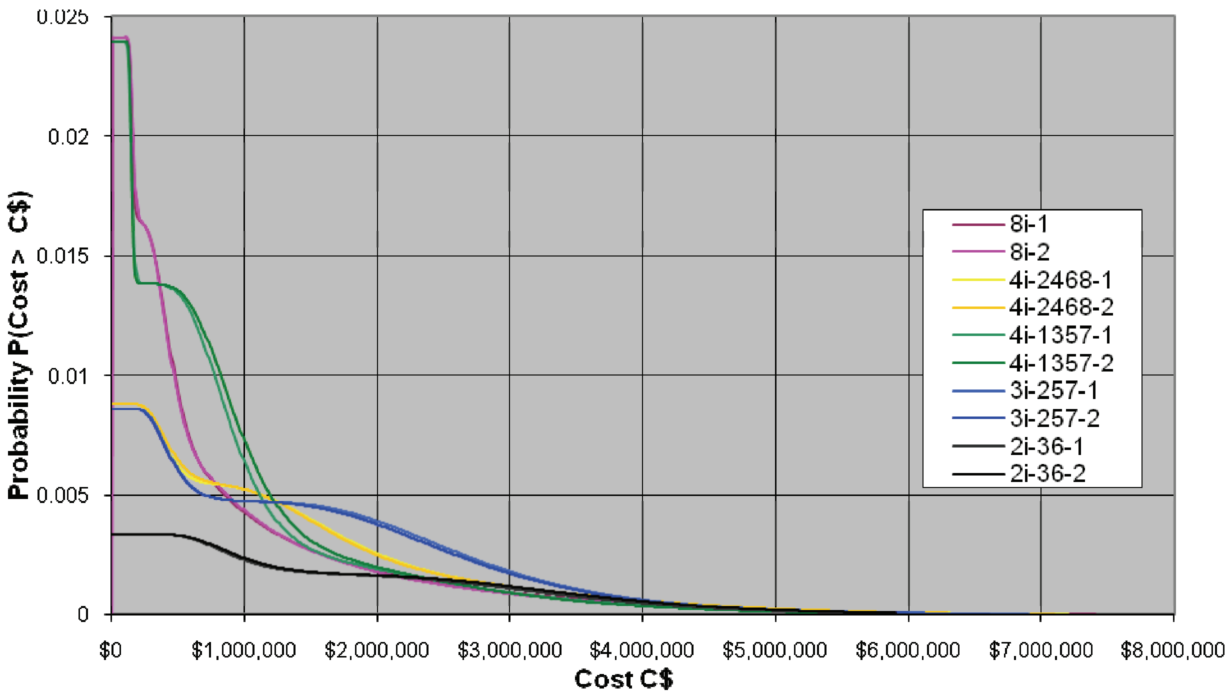

Figure 11. Time-based analysis: Probability of exceedance of repair cost between ($0 - $8 Million)

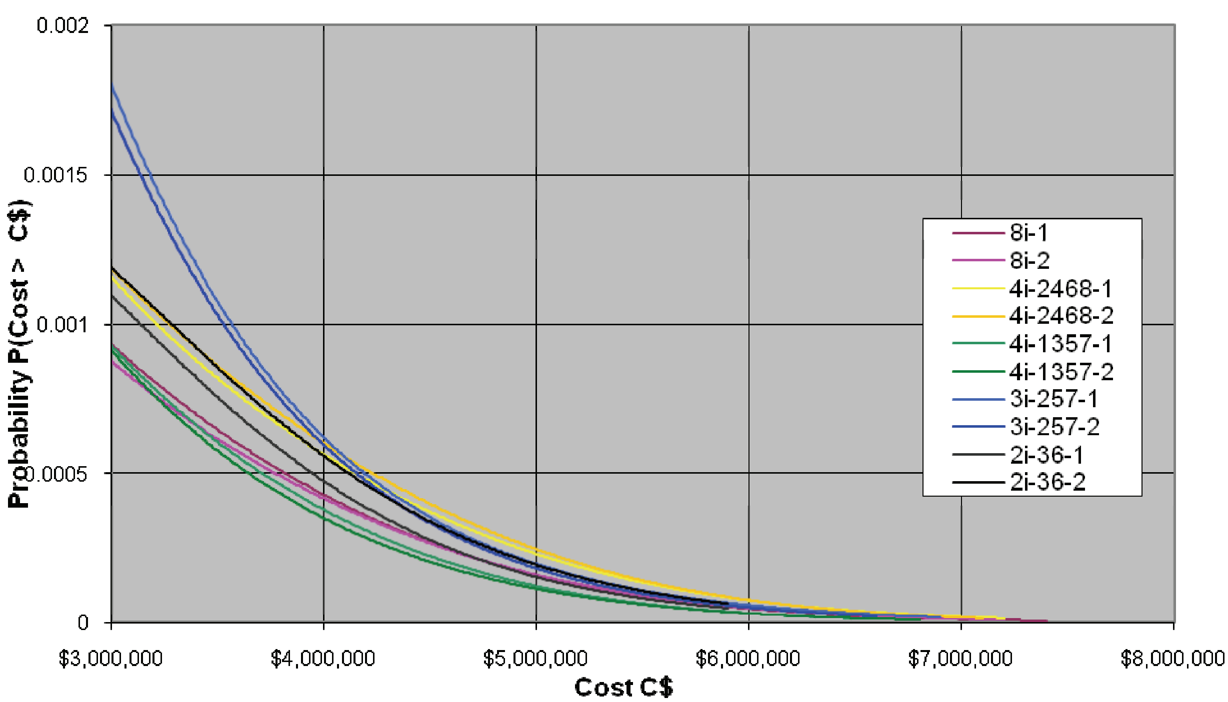

Figure 12. Time-based analysis: Probability of exceedance of repair cost between ($3 - $8 Million)

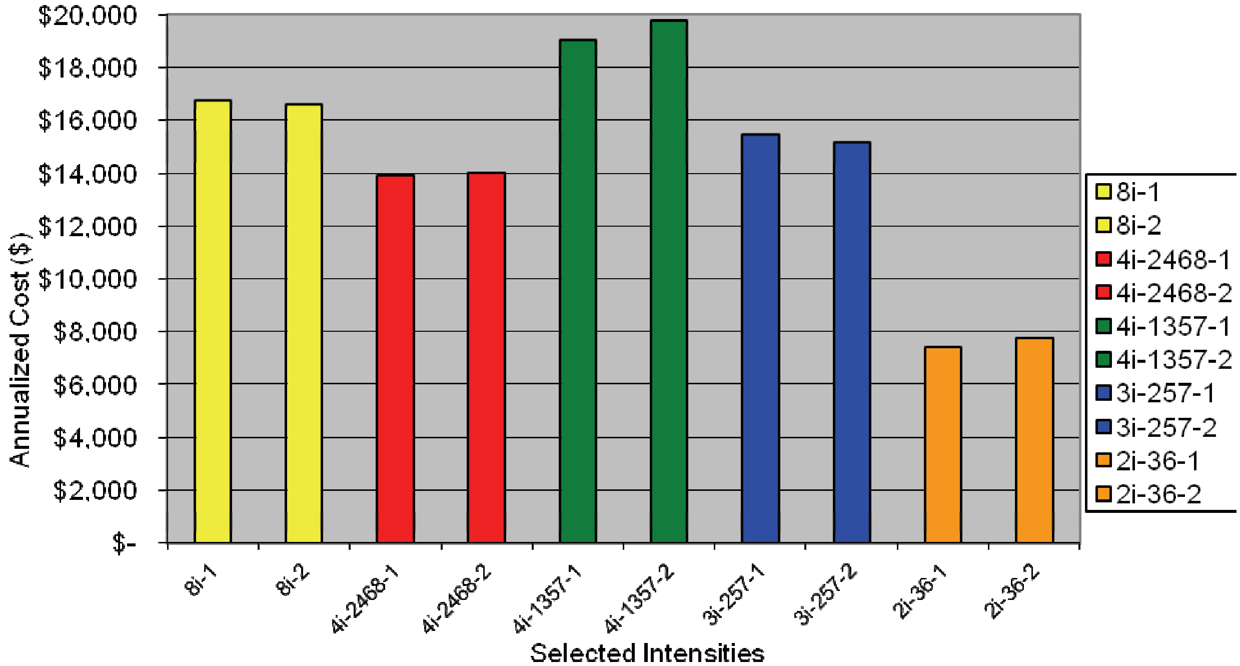

Figure 13. Annualized cost ($) for different number of intensities.

PACT Time-Based Integration Algorithm

The time-based annual loss integration algorithm that is currently implemented in PACT was reviewed. It was found that the current algorithm omits contributions from the first and last point of the hazard curve. While this does not cause a significant difference for probability of exceedance at intermediate probabilities and costs of repair, it can result in a significant deviation near the ends of the curve, i.e. at very high and very low mean annual frequencies of exceedance (MAFE). In addition, this introduces a difference in the annualized cost of repair, which can be highly influenced by contributions from very frequent events, or high MAFE. The effects of including the tails of the curve in the integration are illustrated in Figure 14 and Figure 15.

The PACT development team is aware of this issue, and will address it over the next phase. For the purpose of the studies performed in this report, the tail contribution is not included in the results, since the available PACT version did not include it. It is expected that including the tail contribution in the analysis will not change the conclusions of the parametric analyses performed in this study. Nonetheless, a repeat of some of these analyses is recommended once the PACT integration algorithm is updated.

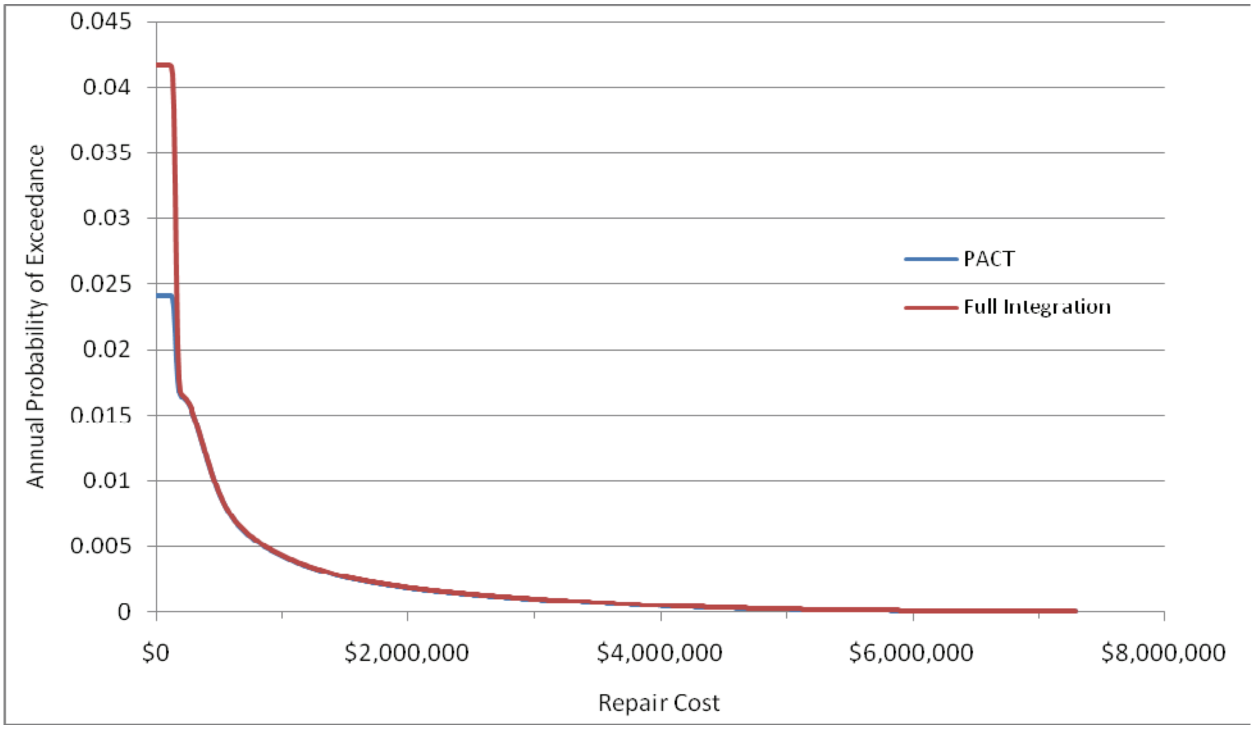

Figure 14. Annual probability of exceedance with and without tail contribution (linear scale)

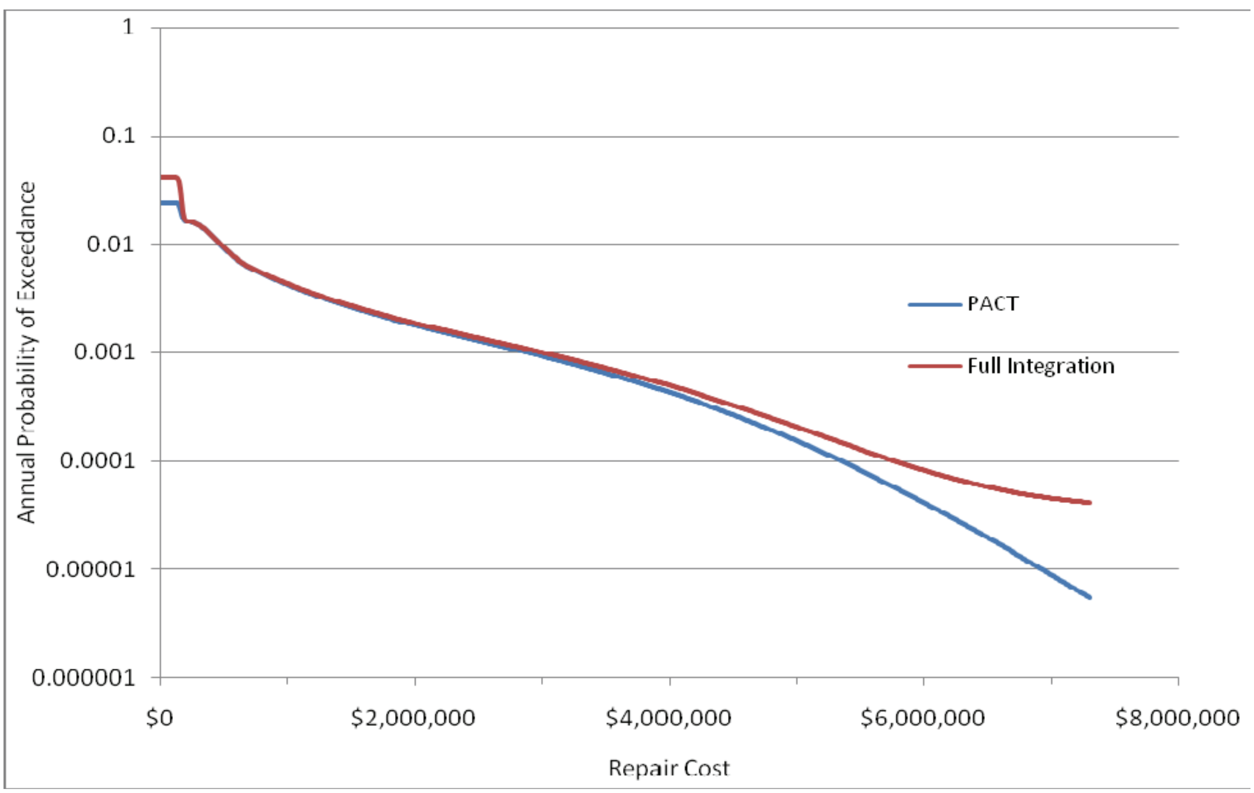

Figure 15. Annual probability of exceedance with and without tail contribution (log scale)

NUMBER OF REALIZATIONS FOR LOSS COMPUTATIONS

This parametric study investigated the effect of the selected number of realizations (N_r) in PACT on the results of the scenario or intensity-based losses. The analysis was performed using the special steel moment-frame office building example that is included with PACT. The purpose of these studies was:

- Verify that PACT provides consistent loss estimates, i.e., the loss value should not vary significantly between runs, given a reasonable number of realizations is used.
- Estimate a reasonable number of required realizations that results in a good confidence in the result values.
- Confirm that the estimated losses converge to a theoretical value as more realizations are used.

Because of the stochastic nature of the loss model, each invocation of PACT is expected to result in different loss calculation values even when the input is similar. In order to quantify this variability, the loss calculation was repeated 25 times for each setting of the solution parameters. This number was judged to be large enough to capture the overall variation in the results, but small enough for the scope of this study. Hence, 25 separate loss calculations were performed for each setting of N_r. Loss values were obtained for each of the eight levels of intensity that were provided in the example, and the statistical distribution of the 25 loss calculations was analyzed by computing the mean, minimum, maximum and standard deviation values. The resulting statistics were compared both by N_r and by intensity as shown in Figure 16 through Figure 25. Statistical values are compared to the mean loss obtained with N_r=10000.

The variability in the estimated losses is clearly reduced as more realizations are used. For example, with 500 or more realizations, the coefficient of variation is less than 2% for all considered intensity levels. The mean loss for each analysis generally appears to slightly decrease with increasing number of realizations. Figure 26 shows that the variability in the computed loss decreases significantly with increasing number of realizations for all intensity levels.

In general, it appears that selecting a number of realizations between 200 and 1000 yields a reasonable degree of accuracy, and that a larger number is not justified. However, since the calculations are reasonably fast, a larger number (example 10,000) may be used to perform a final analysis if desired. A calculation with 10,000 realizations took about 5 minutes on a laptop computer for the considered problem.

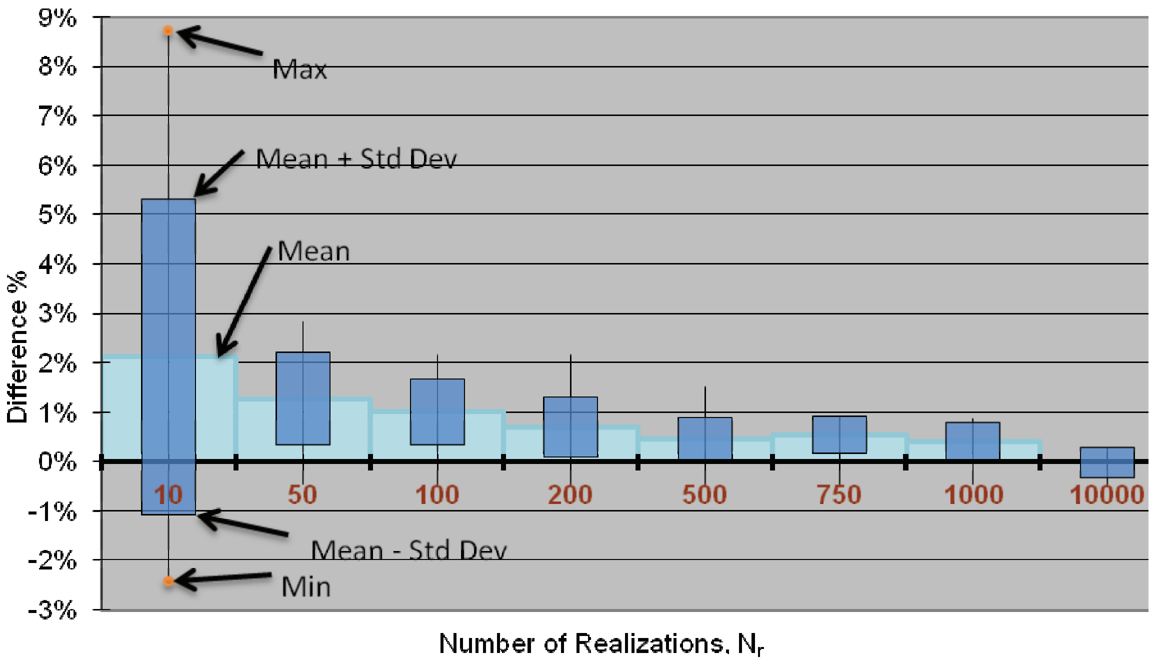

Figure 16. Median loss estimate for Analysis 1: Dispersion of repair cost vs. number of realizations (25 PACT Simulations). Values presented relative to mean estimate using N_r of 10000.

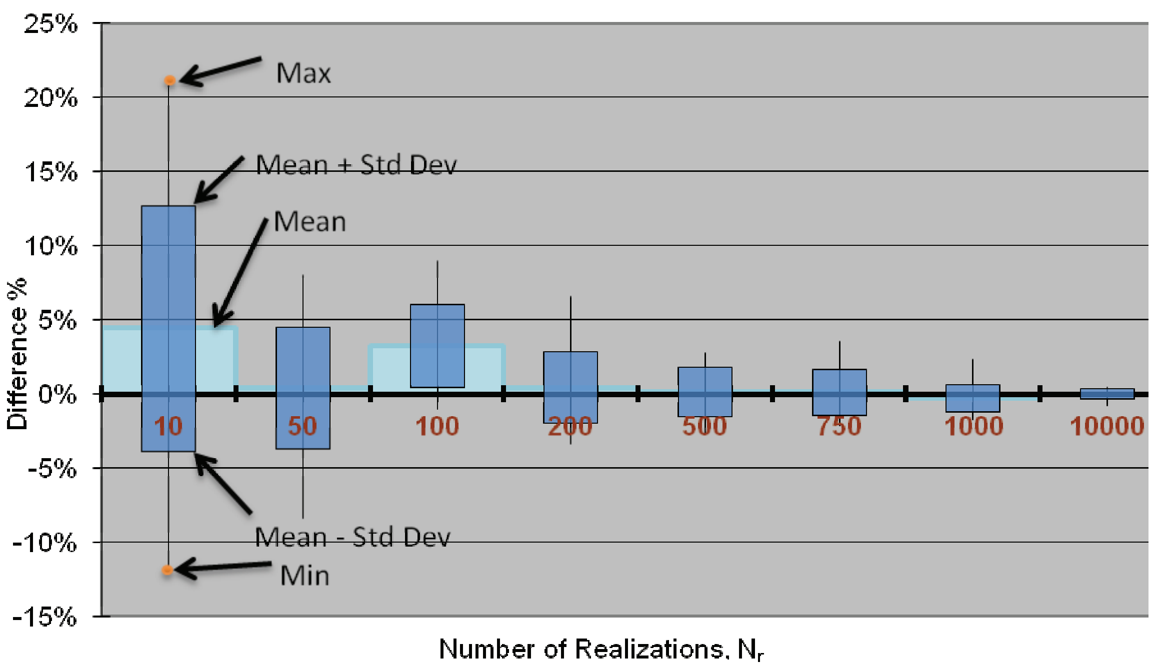

Figure 17. Median loss estimate for Analysis 2: Dispersion of repair cost vs. number of realizations (25 PACT Simulations). Values presented relative to mean estimate using N_r of 10000.

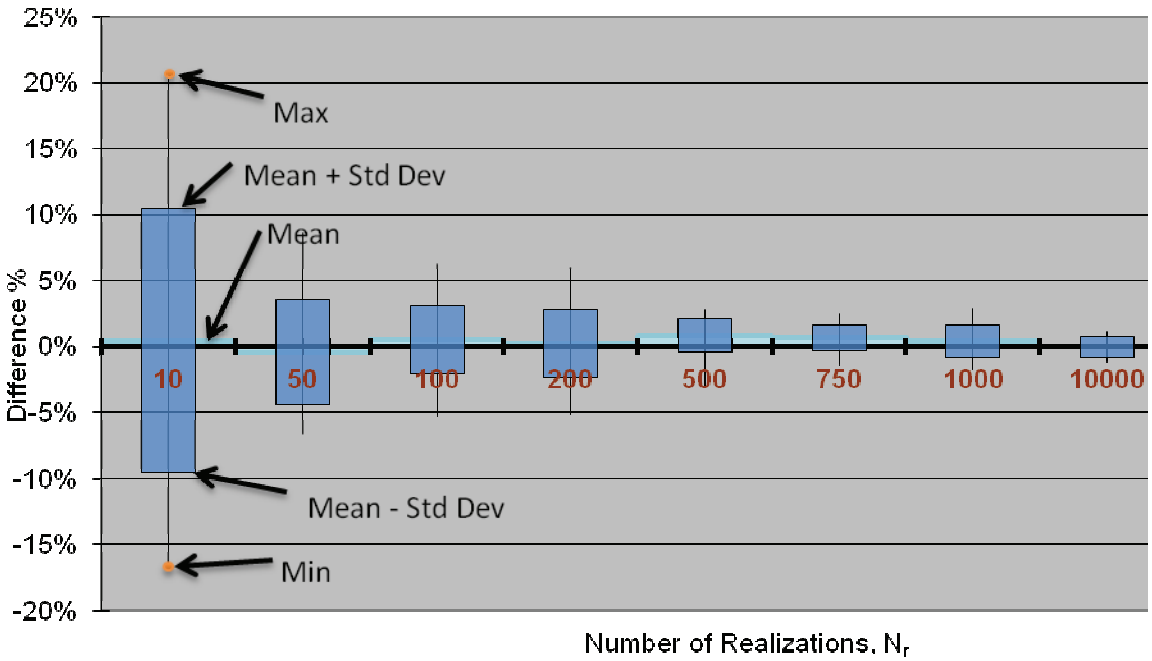

Figure 18. Median loss estimate for Analysis 3: Dispersion of repair cost vs. number of realizations (25 PACT Simulations). Values presented relative to mean estimate using N_r of 10000.

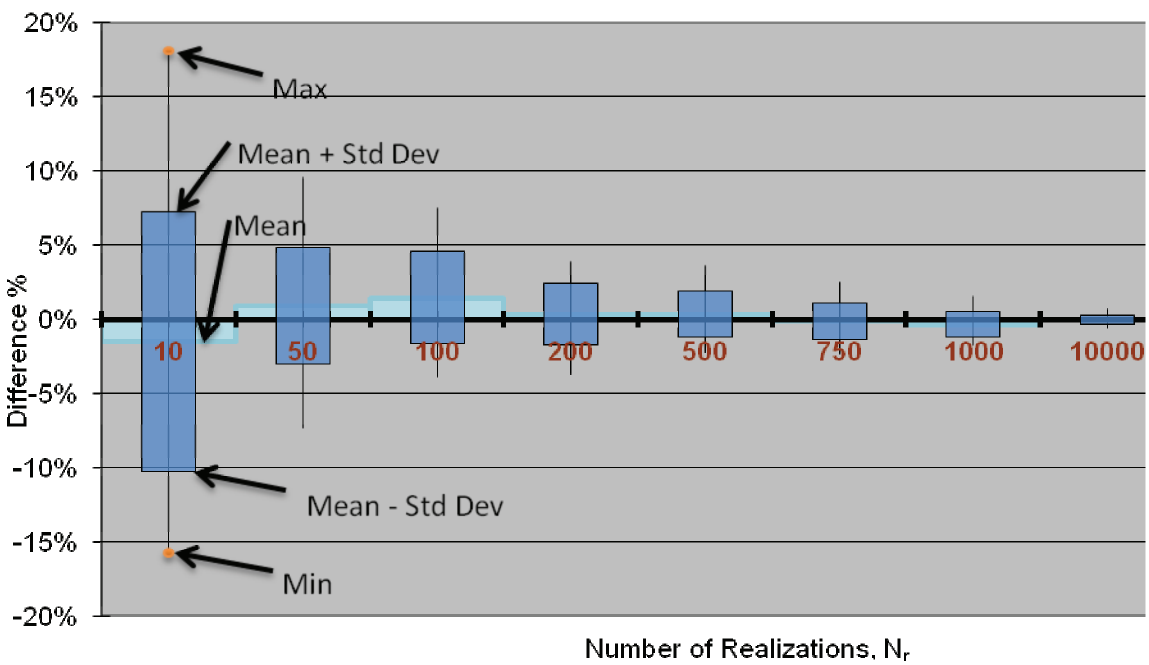

Figure 19. Median loss estimate for Analysis 4: Dispersion of repair cost vs. number of realizations (25 PACT Simulations). Values presented relative to mean estimate using N_r of 10000.

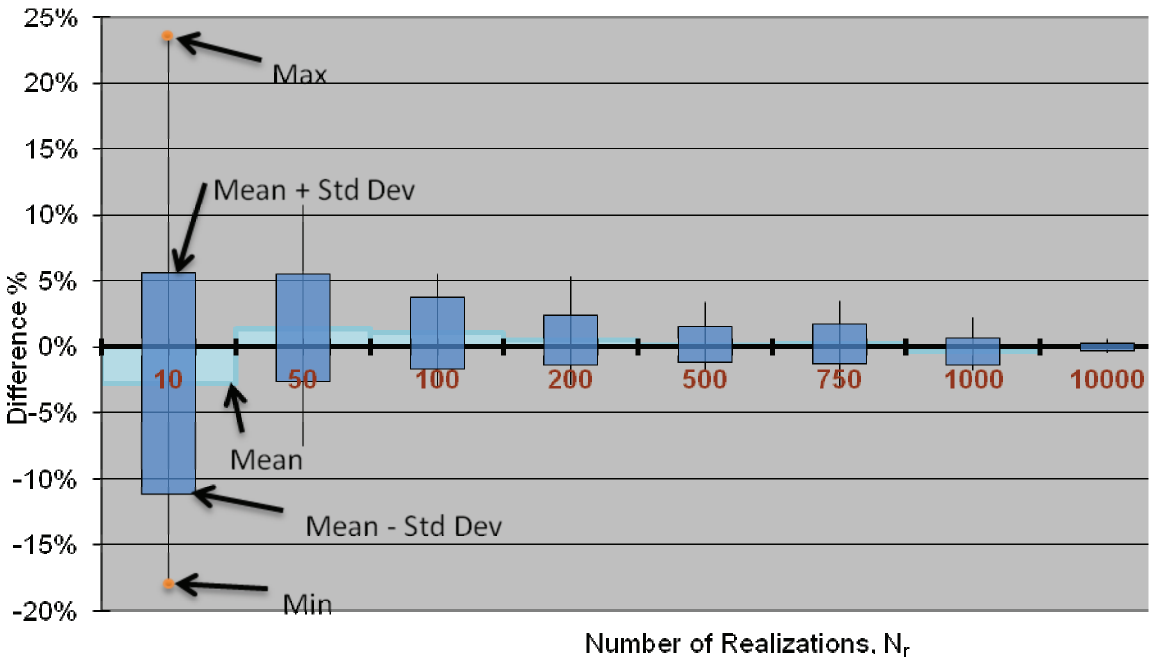

Figure 20. Median loss estimate for Analysis 5: Dispersion of repair cost vs. number of realizations (25 PACT Simulations). Values presented relative to mean estimate using N_r of 10000.

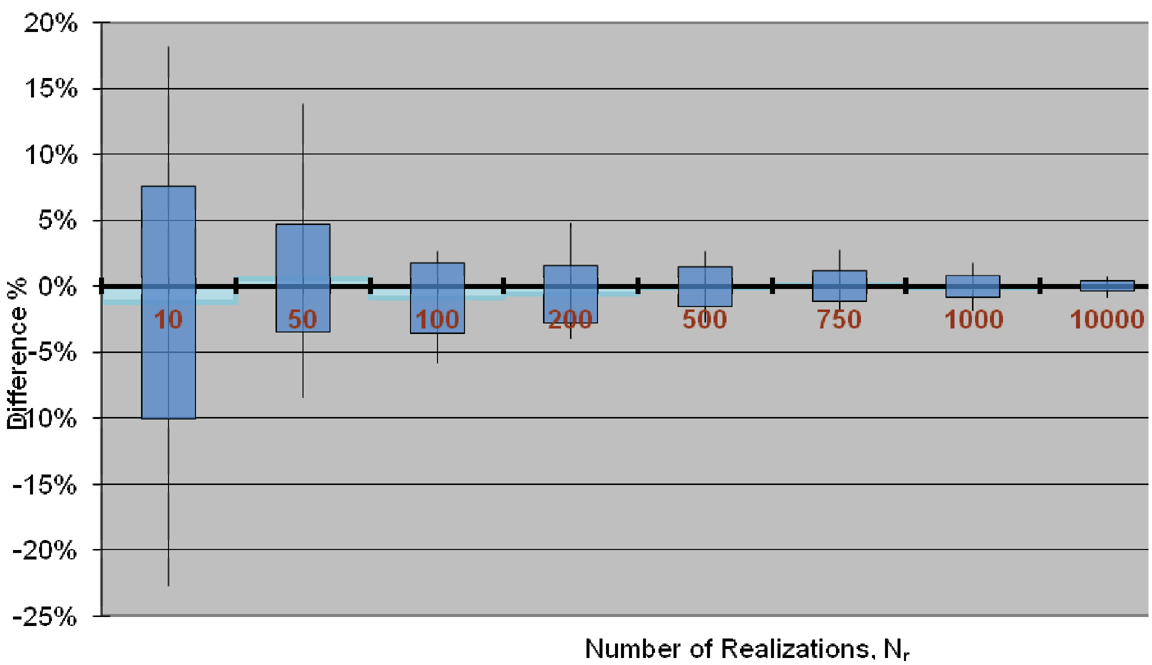

Figure 21. Median loss estimate for Analysis 6: Dispersion of repair cost vs. number of realizations (25 PACT Simulations). Values presented relative to mean estimate using N_r of 10000.

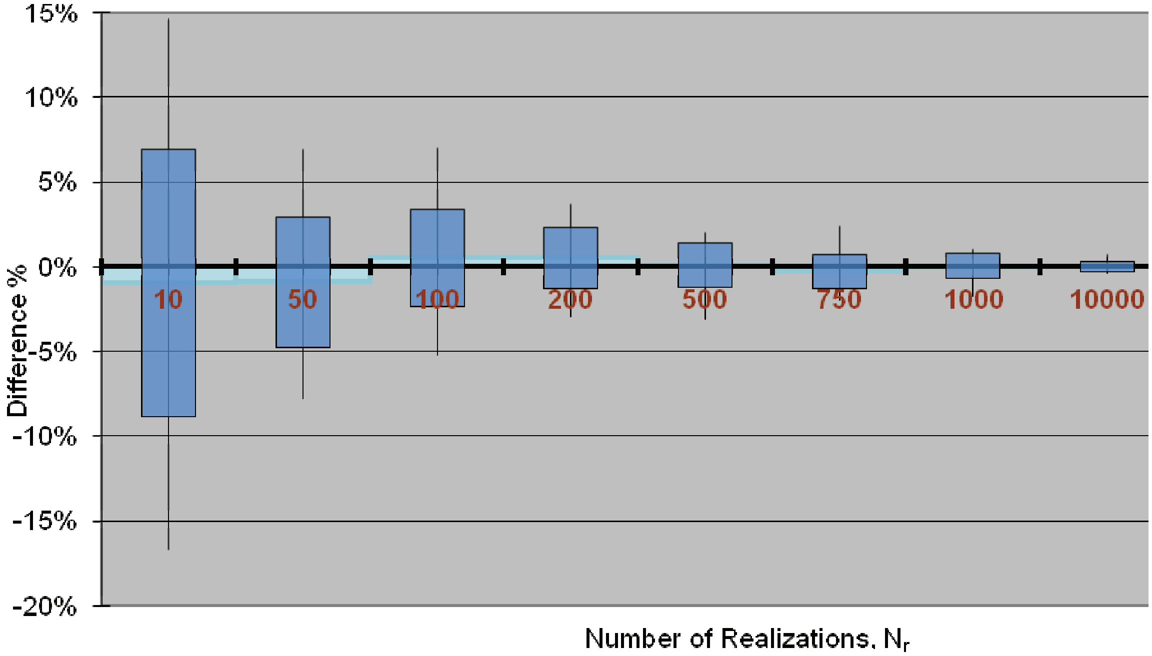

Figure 22. Median loss estimate for Analysis 7: Dispersion of repair cost vs. number of realizations (25 PACT Simulations). Values presented relative to mean estimate using N_r of 10000.

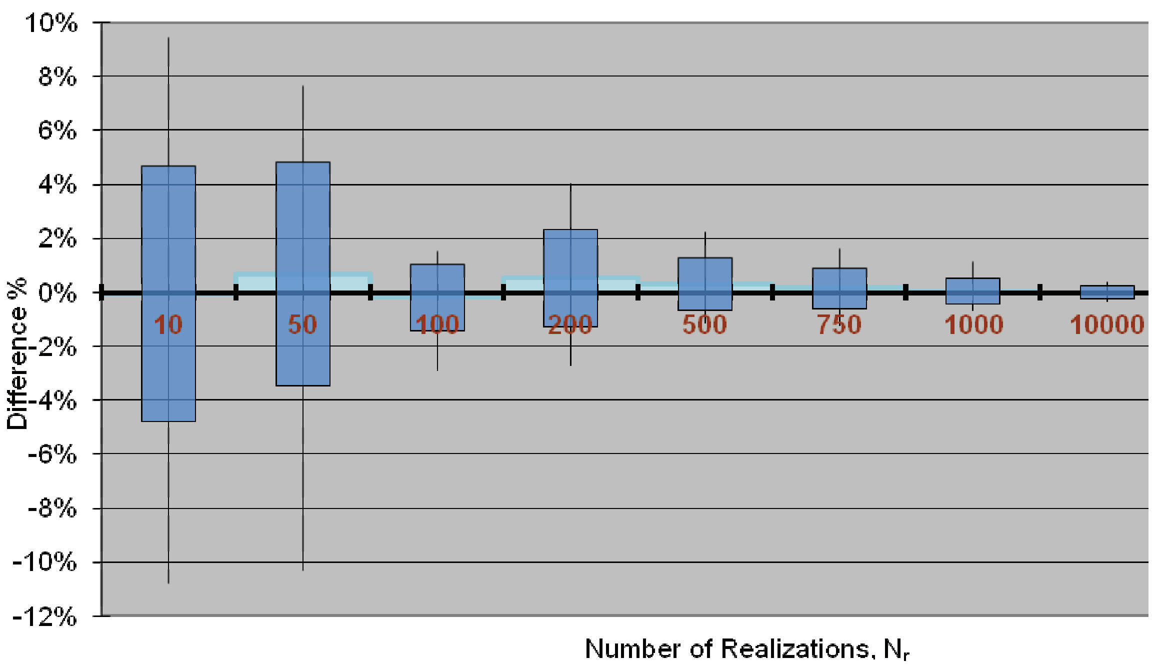

Figure 23. Median loss estimate for Analysis 8: Dispersion of repair cost vs. number of realizations (25 PACT Simulations). Values presented relative to mean estimate using N_r of 10000.

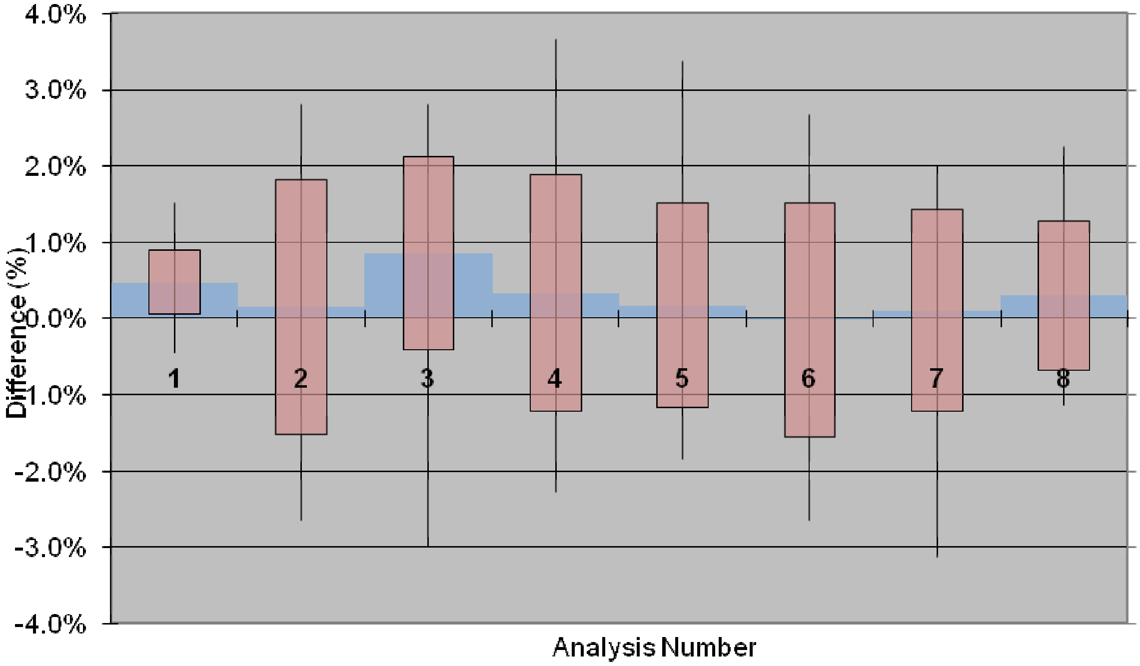

Figure 24. Median loss estimate for N_r=500: Dispersion of repair cost for each intensity level (25 PACT Simulations). Values presented relative to mean estimate using N_r of 10000.

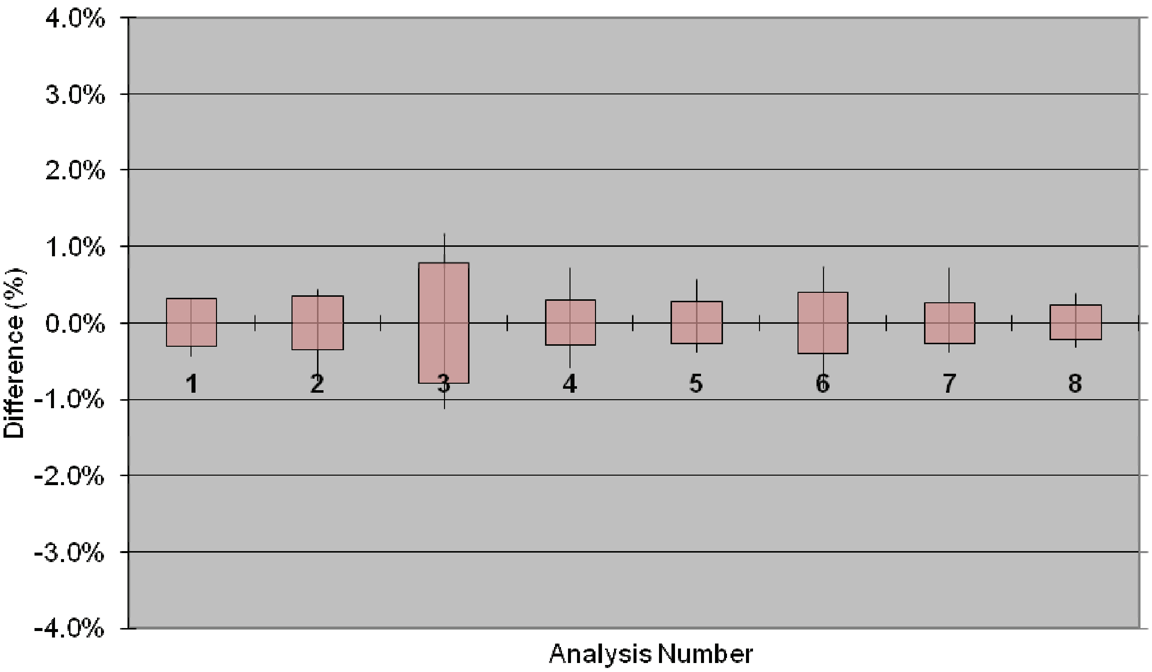

Figure 25. Median loss estimate for N_r=10000: Dispersion of repair cost for each intensity level (25 PACT Simulations).

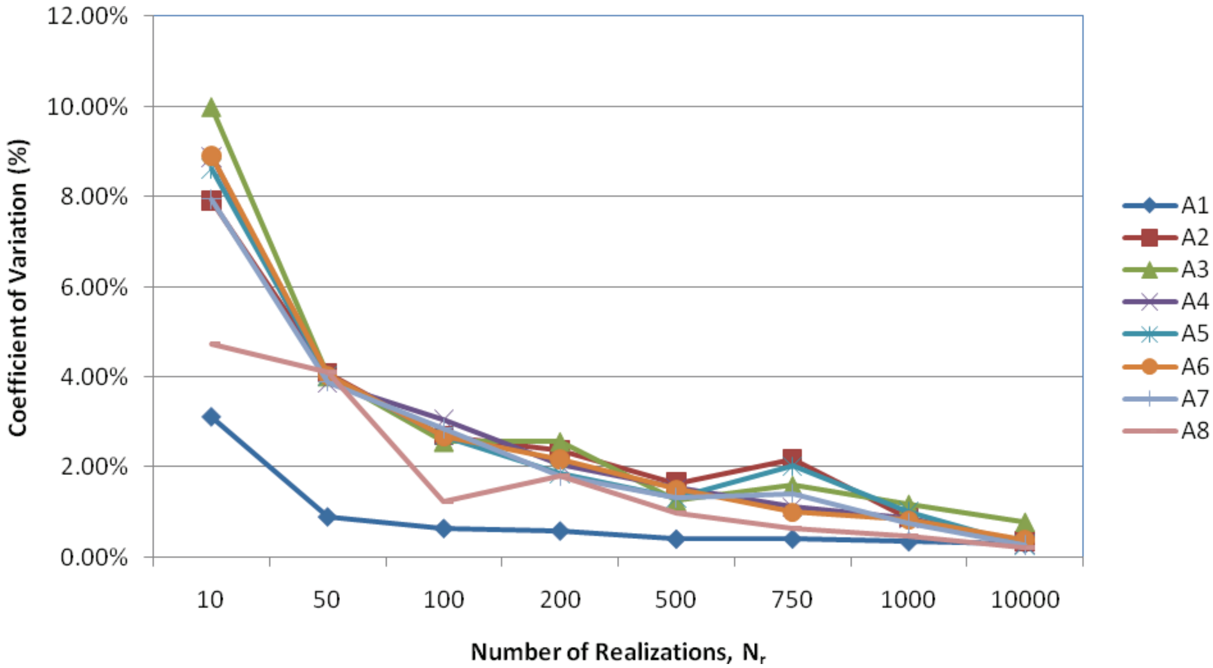

Figure 26. Coefficient of variation (for 25 simulations) vs. number of realizations for each of the eight analyses (increasing intensity levels).