Guidance for Flood Risk Analysis and Mapping

Coastal Statistical Simulation Methods

November 2023

FEMA

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Requirements for the FEMA Risk Mapping, Assessment, and Planning (Risk MAP) Program are specified separately by statute, regulation, or FEMA policy (primarily the Standards for Flood Risk Analysis and Mapping). This document provides guidance to support the requirements and recommends approaches for effective and efficient implementation. Alternate approaches that comply with all requirements are acceptable.

For more information, please visit the FEMA Guidelines and Standards for Flood Risk Analysis and Mapping webpage (https://www.fema.gov/guidelines-and-standards-flood-risk-analysis-and-mapping). Copies of the Standards for Flood Risk Analysis and Mapping policy, related guidance, technical references, and other information about the guidelines and standards development process are all available here. You can also search directly by document title at https://www.fema.gov/resource-document-library.

Table of Revisions

Affected Section or Subsection	Date	Description
Section 1	November 2023	General updates based on additions made to section 3, with regard to the use of parametric and non-parametric fitting method to analyze storm sets.
Section 3	November 2023	Renaming of section to capture larger topic discussion. Addition of expended discussion of use of distribution fitting methods for extra-tropical storms and past use for tropical storms.
Throughout Document	November 2023	General style and document reference updates

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1. Topic Overview

This guidance document supports the standards related to statistical simulation methods. In studies where flood levels are determined by numerical modeling of the physical processes, this is done by simulating a large number of storms over a long period of record and then deriving flood statistics from that simulation. Recent FEMA Flood Map Project flood statistics have been derived through three types of simulation methods involving storm parameterization and random selection: the Joint Probability Method (JPM) (Benjamin and Cornell, 1970), distribution fitting methods -the Empirical Simulation Technique (EST), and the Monte Carlo method., and are all described below, but the focus is on the JPM approach, which has been widely used in recent FEMA studies. Distribution fitting methods are divided into non-parametric methods, for example the Empirical Simulation Technique (EST) and parametric methods.

In areas where both tropical cyclones and extra-tropical cyclones contribute significantly to the overall flood risk, such as on the US East Coast, the analyses of tropical cyclones and extratropical cyclones are typically conducted separately, and the results are then merged to yield the combined coastal flood statistics. (FEMA, 2023). Within this framework, the Joint Probability Method (JPM) has emerged as the preferred approach for tropical cyclone analysis, and distribution fitting has been commonly applied for the analysis of extratropical storms.

Tropical storms are relatively rare events compared to extra-tropical storms but have a common and distinct structure. Storm parameters characterizing the storm structure can be defined, and analysis of historic cyclone data yield the joint probability distributions needed to support the JPM. The distribution fitting methods, and more specifically the EST, are highly sensitive to individual storms. The EST method has been applied to the analysis of Tropical Cyclones in earlier post Katrina studies but has been found to generally be inferior to the JPM for analysis of tropical cyclones in the United States (Divoky & Resio 2007, Resio et al. 2008).

Extratropical storms occur more often, but they do not have a distinct structure that is easily characterized. In this regard the application of the JPM is not feasible. Consequently, the distribution fitting methods are commonly applied for analysis of extratropical storms.

Although the common practices over the past decade have yielded these preferred approaches, all methods are acceptable presuming they appropriately parameterize coastal flood probabilities and should be considered in the context of the available data. If other methods become available, they should be evaluated for their ability to provide accurate estimates of flood statistics and their use should be discussed with the FEMA Project Monitor.

The state-of-the-practice for the JPM is given much more coverage in this document than the other methods as it is the recommended approach. The discussion of the JPM approach is followed by description of the EST applied to tropical storm analysis, and then distribution methods applied to extra-tropical storm analysis are discussed and finally a general discussion of the Monto-Carlo method applied to tropical storm analysis is provided.

2. Joint Probability Method (JPM)

In recent years, it has been recognized that of the presently available methods, JPM is preferred for the tropical storm (or hurricane) environment. The JPM approach has the conceptual advantage of considering all possible storms consistent with the local climatology, each weighted by its appropriate rate of occurrence. In brief, the most basic JPM approach adopts a parametric storm description involving hurricane descriptors such as central pressure, size, and translation speed. For each of the several parameters, probability distributions (not necessarily mutually independent) are developed through an analysis of the local climatology. These distributions are each discretized into a small number of representative values, and wind fields are generated using all possible parameter combinations at multiple locations along the coastline resulting in thousands of potential storm wind and pressure fields. Each individual wind field has its own probability weight which is derived from the parameter distributions. Each individual wind field can then be simulated using a hydrodynamic model constructed to faithfully represent the bathymetry, topography, and ground cover of the study site.

Capturing the full range of physical mechanisms controlling flood levels in the JPM can impose a heavy computational burden on the analyses. The hurricane central pressure (storm intensity), pressure radius (storm size), forward speed, and heading are the 4 most basic hurricane parameters, which should be represented. Holland's B, which reflects the radial distribution of the pressure and wind field, may also be important. This results in 4-5 separate parameter distributions, which can yield thousands to tens of thousands of individual combinations and landfall locations. Details on these and other storm parameters are covered in subsequent subsections.

Because the number of storms grows rapidly with the number of JPM parameters and number of discretizations, it can be desirable to limit this. Some studies, such as the FEMA Mississippi study (FEMA, 2008) have represented Holland's B as part of the uncertainty model. In addition, FEMA and USACE efforts (FEMA 2008; Resio 2007) independently developed new and highly efficient methods of implementing the JPM approach in such a way as to minimize the number of storms requiring simulation. It was found that the simulation effort could be reduced by an order of magnitude or more. Doing so yielded stillwater level results that differed from those of the full JPM set by less than the uncertainty in the hydrodynamic and meteorological models being used, making it a worthwhile trade-off. . The two approaches are both known as Optimal Sampling methods (OS), denoting their common intent of choosing storms for simulation in such a way as to accurately cover the entire storm parameter space through optimal parameter selection with associated weighting and interpolation methods. The full JPM and JPM-OS methods are discussed below.

2.1. JPM Overview

A JPM storm surge flood study of a regional area, such is typically done for recent FEMA storm surge studies, requires two types of knowledge: first, the Mapping Partner requires a knowledge of the storm climatology of the study region in order to be able to characterize the storms governing the coastal flood hazard; second, the analyst requires a knowledge of the effects produced by a particular storm throughout the study region. The former type of knowledge is obtained by a study of the storm history within the vicinity of the study site. The second type of knowledge is provided by use of a validated hydrodynamic model capable of simulating the details of flooding for any storm affecting the region.

Knowledge of the local storm characteristics introduces two problems of practical importance, which will be seen to color much of the JPM-OS discussion to follow. Knowledge of the local storms is based on local storm data. However, data must usually be taken from outside the immediate study area in order to obtain a sample of reasonable size upon which statistics can be based; a particular county, for example, may not have been the site of hurricane landfall within the entire historical record. This raises the question of how far afield one can go in assembling a sample: clearly, storms from distant points may not be of the same character as local storms. So the unavoidable problem is to balance sample error on the one hand, versus population error on the other. Small samples are subject to large variability in a random historical record, while a larger sample from distant points may be corrupted with storms unlike those belonging to the local population.

The role of the JPM is to create a computation scheme that considers all possible hypothetical storms to be constructed from a small number of storm parameters embedded in a storm model of winds and pressures. Current practice is to consider enough defining parameters as sufficient to specify an idealized storm. The following paragraph discusses some of the commonly relevant parameters. Figure 1 shows a characterization of a storm as it approaches the coast with parameters such as the direction of storm motion, θ, a track location parameter, such as by-passing distance, d, the speed of storm translation, V_f, a measure of intensity such as the central pressure deficit, ΔP, and a measure of the storm size such as the radius to maximum winds, R_{max}, or the pressure scaling radius, R_p. Figure 1 does not show the Holland B parameter, which changes the shape of the wind and pressure fields and affects the maximum wind speed for a given pressure deficit.

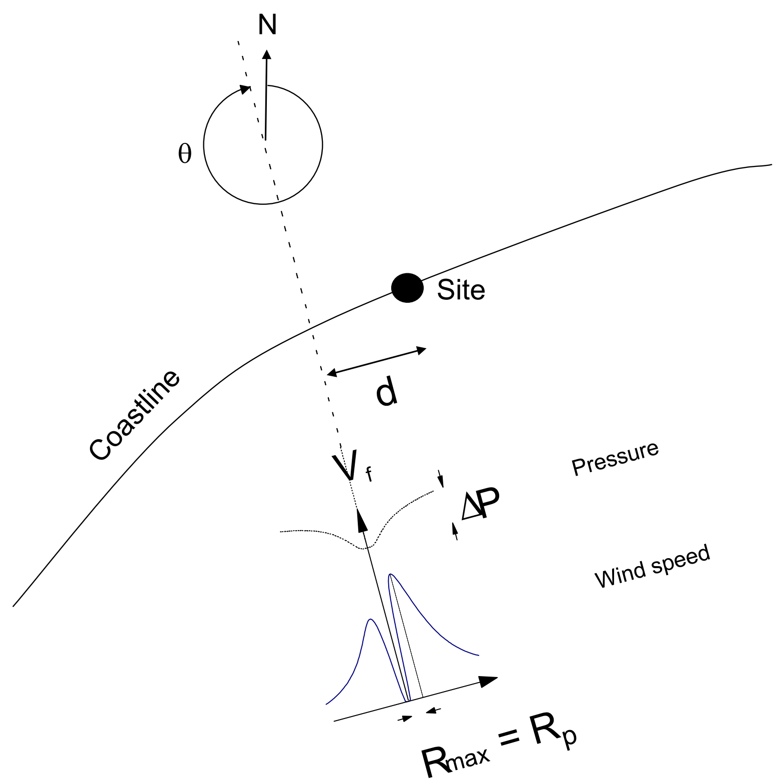

Figure 1: Characterization of a storm as it approaches the coast with commonly relevant storm parameters

Storm strength is characterized by the central pressure deficit, or the difference between the pressure at an assumed storm eye and the ambient pressure at the storm periphery. Storm size is measured by some length parameter which approximates the radial distance from the eye to the zone of maximum wind speed. The relative sharpness of the peak of the radial pressure profile may be controlled by a fitting factor (Holland's B). The storm track, in its simplest straight-track form, might be characterized by three kinematic parameters: the direction of storm motion, a shoreline crossing point (or bypass distance), and a speed of storm translation. Many other parameters could be added to this mix. For example, storm surge occurs in superposition with astronomic tide; consequently, tide amplitude and phase might be enlisted as additional parameters. Storm tracks are not straight, and speeds are not constant. Storms are not simple circular affairs accurately captured by idealized analytical forms of radial profiles. Therefore, any number of higher-order parameters could be invoked to permit one to capture the possible range of real events, but each additional parameter will increase the number of model storms needed. In this document, only the basic parameters are addressed.

Working, then, with a relatively small set of parameters, the JPM procedure generally proceeds as follows (using numbers chosen for illustration only, not for guidance):

- First, develop probability distributions for each storm parameter. The probability distributions are derived from a storm sample which, in its simplest form, can be thought of as a list of all storms and their parameters recorded during a selected period, within a region surrounding the study site. Familiar distribution forms are fit to the data as appropriate. These empirical distributions need not be independent. For example, the distribution of storm size is commonly taken to be conditional upon the central pressure deficit, so that stronger storms tend to be associated with smaller radii. More significantly, all other parameters are always taken to be conditional upon track direction in the case of a site that is affected by both entering and exiting storms (such as the Florida peninsula).

- Second, establish the overall rate of storm occurrence in both space and time. In a sense to be made more precise later, let this be the number of storms passing per unit length of space per unit time; storms per mile per year, for example (typically a small number).

- Third, for a basic JPM (not-OS) study, subdivide each distribution into a small number of discrete pieces; one might imagine representing pressure, radius, forward speed, and track angle by a half dozen, or so, values of each.

- Fourth, construct all possible hypothetical storms by simply taking all possible combinations of these elementary storm quantities. With, say, six values for each of the four parameters mentioned in Step 3, above, one constructs 1296 “storms.” These storms constitute the simulation set.

- Fifth, simulate all of these storms, each on multiple tracks so as to allow every storm type to affect all points in the study area. That is, a particular storm may pass through any point along the coastline of the site, so that random track position must be accounted for. In the simplest case, this might be done by adopting a track spacing dependent upon the storm size, and replicating the tracks for a particular storm by parallel displacement. Usually, for small study sites, track position is distributed uniformly over space.

- Sixth, for each such storm determine a corresponding rate of occurrence. This is just the product of (1) the overall rate of occurrence from Step 2, above; (2) the probability masses of each of the four parameter chunks from Step 3 (reflecting dependence as appropriate); and (3) the selected spacing between tracks adopted in Step 5. That is, each simulated track is taken to represent all possible tracks which could occur over a zone extending half way to its neighboring tracks on each side. The track spacing is chosen small enough to provide smooth coverage of the site; tracks spaced too far apart will produce a fluctuating surge estimate, underestimating the potential at points between the simulated surge peaks. On the other hand, tracks spaced too close together would necessitate an excessive number of simulations, imposing an unnecessary computational cost. It has been found that track spacing equal to the radius of maximum winds provides good results in the FEMA Mississippi study (FEMA 2008, Coastal Documentation and Main Engineering Report Section 3.3.10). Since storm radii may be only a few tens of miles while the site may extend for a hundred or more miles, each storm must be simulated on several tracks in order to cover the area (the number of tracks will be greater for the small storms in the simulation set than for large storms). In practice, ten or more tracks per storm may be required.

This brings the number of storm simulations in the example to 10 ×1296, or nearly 13,000 model runs, each requiring a number of hours.

- Seventh, at each point of interest in the hydrodynamic grid – the target sites selected for the final statistical analysis and the mapping effort – record the highest surge computed for each storm, and tag it with the rate of occurrence of that storm.
- Eighth, for each target site, construct a histogram of rate versus surge height, by accumulating the storm rates into the bins of surge height. Such bins might be constructed with widths of 0.1 ft., for example, so that a histogram with bins running from 0 to 500 would handle surge elevations up to 50 feet. The accumulated rates in the bins constitute an estimate of the *density distribution* of surge height.
- Ninth, for each histogram (one for each target site) sum the rates from the top bin down to the bottom bin. The result of this step is an estimate of the cumulative surge distribution. To find the 1% surge elevation, for example, one simply locates the bin having a summed rate nearest 0.01. For example, if this occurs at bin 232, the estimate of the 1% surge elevation would be 23.2 feet, following the assumptions made in this example.

The sections to follow develop the JPM-OS approach in a more rigorous way than outlined here. The essential difference between the foregoing bullets and the OS method, to be described below, is in Steps 3 and 4, above. Rather than constructing the storm simulation set by simply subdividing each parameter distribution into a small number of chunks and simulating all possible combinations, the OS approach is to select the storms (combinations of parameters) and their weights in a much more intelligent way, so as to reduce the computational burden by about an order of magnitude.

2.1.1. SUMMARY OF THE JPM APPROACH

The JPM method is now summarized in more formal terms in preparation for the subsequent discussions. As noted above, the approach relies on probabilistic descriptions of storm occurrence and storm characteristics to define a set of synthetic storms, together with a numerical method to calculate the coastal flood elevations that would be generated by those storms. The numerical method includes representations of the storm tracks, the evolution of storm characteristics (referenced to the characteristics at landfall), the wind and pressure model, the surge model, and so forth, represented symbolically as

$$\eta(\Delta P, Rp, Vf, \text{landfall location}, \theta, \ldots) = \eta(X) \qquad (1)$$

where η is the surge elevation at a point and the vector *X* represents all pertinent storm characteristics including the central pressure deficit, ΔP; the storm radius, R_p; the forward speed of storm motion, V_f; the storm track direction, θ; landfall location; and others as may be of significance such as Holland's B parameter, astronomic tide factors, and so forth. The landfall location and track angle determine the proximity of the storm to a particular coastal site. The annual rate of occurrence of a flood elevation at the site in excess of η is defined in terms of the combined probabilities of the storm parameters and is given by the multiple integral:

$$P[\eta\max(1\ yr) > \eta\ ] = \lambda \int \dots \int_{\underline{x}} f_{\underline{x}}\ (\underline{x}) P[\eta(\underline{x}) > \eta] d\underline{x} \qquad (2)$$

where λ is the mean annual rate of all storms of interest for that site, $f_{\underline{x}}(\underline{x})$ is the joint probability density function of the storm characteristics of these storms, and $P[\eta(\underline{x}) > \eta]$ is the conditional probability that a storm with characteristics $\underline{x}$ will generate a flood elevation in excess of η. This integral over all possible storms determines the fraction of storms that produce flood elevations in excess of the value of interest, using the total probability theorem (Benjamin and Cornell, 1970). The entire expression, including λ, is actually a rate with dimensions of events per unit time, but is commonly thought of as an annual probability to a good approximation.

Evaluation of the JPM integral (Equation 2) by use of conventional brute-force numerical-integration approaches is problematic since each evaluation of the integrand involves the costly evaluation of $\eta(\underline{x})$ for one set of parameters, $\underline{x}$, (that is, the simulation of one storm), and since the evaluation of the 5-dimensional (or higher) integral in the equation requires that the integrand be evaluated a very large number of times.

2.1.2. JPM PARAMETER SELECTION

As suggested above, the JPM approach adopts a parameterized representation of tropical storms, which at a minimum usually involves:

- a measure of intensity: the central pressure deficit, ΔP (usually given in millibars).
- a measure of storm size: the radius to maximum winds, R_{max}, or the pressure scaling radius, R_p (usually given in kilometers).
- the speed of storm translation, V_f
- the direction of storm motion, θ (direction of motion typically measured counterclockwise from north)
- a track location parameter, such as the shoreline crossing point X_c or a by-passing distance

Storm parameters should be defined with respect to a specified reference condition; in particular, observed values at landfall (with respect to a nominal shoreline for the study site) should be used as the basis for parameter descriptions.

The number of parameters to be accounted for in a JPM analysis can be increased indefinitely, as greater and greater complexity is added to the description of a storm and additional factors such as rainfall intensity and spatial pattern are included in the list of flood mechanisms. As will be shown in a subsequent section, however, one quickly reaches a limit of what can be treated by simulation of combinations of all parameters owing to the curse of dimensionality. If only five parameters are to be considered, and if each of these is represented by only six values over its significant range, then there would be a total of 6^5 combinations, or 7,776 in all, each representing a synthetic storm. Modern hydrodynamic models for waves and surge might require a number of hours to simulate a single storm even using a high performance computing cluster, so that efforts of this sort are not feasible. The entire goal of the OS variants of the JPM approach is to provide a sufficiently accurate representation of the storm climatology, while reducing the size of the simulation storm set to fall below a feasible limit.

The Mapping Partner must include all important parameters for a JPM study at a site, typically including at least the first five enumerated in the list above. Storm surge and storm likelihood are both known to vary appreciably with respect to each of these parameters, and all major JPM-based studies of hurricane surge since the original NOAA studies of the 1970s (Ho and Myers 1975, Myers 1975) have utilized at least these first five parameters, such that using fewer terms is unlikely anywhere in the United States. But the Mapping Partner should recognize that little can be gained by adding parameters if the available data is not sufficient to develop the required probability distributions. Similarly, it would be unrealistic to simulate an excessive number of parameter combinations to represent a joint probability distribution that is not known well because of data limitations; although resolution/precision might be gained, accuracy would not.

Note that although the distributions of storm parameters will usually be defined in terms of shoreline crossing values (or an equivalent for bypassing storms), the storm simulations may treat the parameters as variable during the course of a simulation. In particular, pressure may be assumed to vary in a prescribed manner both before and after landfall (filling), as may Holland’s B parameter if it is used as a basic JPM parameter. Similarly, while track angle statistics are based on values at landfall, the simulated tracks may be curved in a defined manner to better represent pre-landfall behavior (this may be of importance for representation of wind wave generation, pertinent to the estimation of wave radiation stresses to be included in the surge simulations).

2.1.3. STORM DATA SELECTION

These guidelines are focused on tropical storms, for which the quality of the historical record has varied greatly over time. Furthermore, not all storm parameters are known equally well. For example, kinematic parameters (based on storm location, from which direction and forward speed can be derived) may be known reasonably well for older storms, although the corresponding central pressures, radii, and B factors, may be absent or known only very approximately. This variability of the quality of the record and disparity in data availability require the Mapping Partner to begin a study with a careful review of data sources.

Following the precedent of the post-Katrina FEMA (FEMA, 2008) and US Army Corps of Engineers (USACE) studies (Resio 2007), it is typically recommended that flood studies for tropical storms be based primarily on data recorded since roughly 1940. This corresponds to the modern era of aircraft reconnaissance and is thought to be much more reliable than older data, especially for both pressure and radius. Although data regarding the kinematic parameters of older storms may be useful, particularly storm tracks and counts, the Mapping Partner should review them critically before including them in the development of parameter statistics. It is noted, too, that storm counts may be unreliable and underestimated for earlier decades, except for nearshore tracks; some distant storms may have been missed, leading to a misestimate of storm density if counts are made over large areas. However, in cases where older historical records show large deviations from more recent data, such as a significant increase in storm rate, an extension of the period of record may be needed.

Basic data for tracks and pressures (but not including radius or B) can be found in the HURDAT data files maintained by the National Oceanic and Atmospheric Administration's (NOAA's) National Weather Service (NWS) Hurricane Research Center (HRC). Note that while track data should be adequate for JPM studies, pressure data should be checked against other sources, including the more detailed storm summaries compiled by NOAA. Pressures inferred from the National Weather Service's (NWS's) hurricane database (HURDAT) windspeeds (by back-computation from a windspeed vs. pressure formula) should not be relied upon. The HURDAT track data consists of latitude and longitude of the hurricane center at six hour intervals. From these, the necessary data for forward speed and track direction can be determined. HURDAT is the official database of hurricane data for the North Atlantic and Gulf of Mexico, and can be obtained (along with descriptive information defining the database structure) from the HRC webpage at www.nhc.noaa.gov.

Other data sources include a special storm compilation produced for FEMA by NWS (NWS-38) and used as the source of JPM information in earlier FEMA studies. Although not up to date, this document includes valuable information regarding storm radii and pressures which is lacking in the HURDAT database. The Mapping Partner should not, however, adopt the NWS-38 statistical summaries for a new study, but should follow the procedures outlined in these new guidelines.

Other data sources must be searched by the Mapping Partner to supplement HURDAT and NWS-38. Data available from the many NOAA divisions including HRC, National Hurricane Center (NHC), the Hurricane Research Division (HRD), and the National Climatic Data Center (NCDC) should be interrogated. For modern tropical storms, the NHC publishes detailed storm summary papers in their Tropical Cyclone Report series, as well as numerous storm analyses available at www.nhc.noaa.gov.

Additionally, private sources of data exist, which may be sources of data or data analyses not otherwise readily available and should be considered by Mapping Partners. In particular, detailed parameter evaluations and determination of "best winds" and "best tracks" may have been made by such private organizations for storms of interest (note, however, that these parameter selections may be conditional upon other assumptions made regarding wind models, and so should be interpreted carefully for application in a study using different methods).

2.1.4. APPROACHES FOR DEFINITION OF THE SAMPLE STATISTICAL ANALYSIS

Two approaches have been used in recent studies for definition of the hurricane sample and the statistical analysis of the hurricane data.

The approach here called the “capture zone” approach is perhaps the more conventional of the two approaches. In this approach, all hurricanes that make landfall along a particular segment of the coastline are counted and are given equal weight in the calculations. Alternatively, the capture zone might be chosen to consist of a spatial region, such as a circular window surrounding the study site. The definition of the capture zone is extremely important and must be chosen with two competing factors in mind. First, the zone must be large enough to capture a significant number of storms, adequate for estimation of parameter statistics. Second, the zone must be small enough to ensure parameter homogeneity throughout the zone. These conflicting requirements represent the problem of sample error, on the one hand, and population error on the other.

The second approach is here called the Chouinard Kernel Approach (or, more briefly, the Chouinard approach). It was introduced by Chouinard and his co-workers (Chouinard, et al, 1997a, 1997b) for use in mapping the hazard from hurricane-induced waves and winds for the offshore-oil industry but is also appropriate for hurricane surge studies. In this approach, each hurricane is given a weight that decreases as the distance from the hurricane to the point under consideration increases. Thus, data from hurricanes that passed near the point under consideration are given more weight than those that passed far from the point. This technique minimizes population error, by emphasizing events that occurred near the site, while also alleviating sample error by allowing additional data to be taken from a distance. The function used to calculate this weight (the kernel function) is typically a Gaussian probability density function, but other shapes may be used if there is reason for them. When using a Gaussian kernel, the scaling parameter that controls the width of the kernel (the kernel width) is then numerically identical to the standard deviation.

One of the most important steps in the Chouinard approach is the determination of the optimal kernel size, which provides the optimal compromise between high geographical resolution and statistical precision (i.e., low statistical uncertainty). This is effectively the same as the problem of choosing the size of the capture zone in the alternate approach mentioned earlier. Chouinard and his co-workers as well as the post-Katrina FEMA study of Mississippi, used a statistical technique known as cross-validation (to be described below) to determine the optimal kernel size, but other techniques may also be used.

The two main advantages of the Chouinard approach are that (1) it includes an objective procedure to achieve an optimal tradeoff between spatial resolution and statistical precision, and that (2) the weight given to a specific storm -- and, therefore, the calculated statistics -- varies gradually as the site of interest is moved or as the kernel width is varied. The second advantage is particularly important in wave-hazard mapping, but it also avoids the problems that may arise if an important historical hurricane happens to occur near the boundary of the capture zone and so may or may not be included in the sample depending upon a small difference in capture zone size. It is best if the adopted method is not sensitive to fine considerations such as this. The main disadvantage of the Chouinard approach is that it is more complex than the capture zone approach.

There are also two approaches for counting hurricanes for the purpose of rate calculations and for defining the distribution of pressure (and the distributions of other storm characteristics that show a significant geographical variation through the study region). One approach (the point-based approach) considers the minimum distance to the location of interest (which will typically be the mid-point of the coastal segment of interest) and computes the distribution of the corresponding parameters; this is the approach used by Chouinard et al. (1997a, 1997b). For applications on open water, one determines the time of the track's closest approach to the location of interest and uses the values of the parameters which existed at that time. In application to surge calculations, where interest is focused on the hurricane's characteristics at landfall, the track is considered to be linear with the heading it had at landfall. This linearized track is then used to determine the minimum distance to the site of interest, and the storm characteristics at landfall are assumed everywhere along the linearized track. In the point-based approach, one may calculate either the omni-directional rate (storms/km/yr) or the directional rate (storms/km/directional-degree/yr). The directional rate is required for surge calculations and can be obtained directly from the omnidirectional rate by combination with the observed distribution of angles.

The second approach (line based) measures distances along the coastline of the region of interest. This approach appears simple but is dependent on the geometry of the coastline. Even if the coastline is simple enough to be idealized as straight, the directional rate is a function of the offshore directional rate and of the coastline's orientation (the proportionality factor is the cosine of the angle between the track heading and the landward perpendicular to the coastline), while the directional rate obtained using the point-based approach is identical to the offshore directional rate (ignoring any possible effects of the land mass on the geometry of the pre-landfall tracks). In the second approach, then, the distribution of heading must be calculated as an additional step.

The most common methodology is to use the Chouinard approach in conjunction with point-based counting of hurricanes and critical parameters such as the central pressure, and the capture zone approach in conjunction with line-based counting for parameters. In principle, one could use the Chouinard approach with line-based counting and one could use a rectangular kernel of arbitrarily selected size (equivalent to the capture zone approach) with point-based counting. The Chouinard approach is recommended for determination of storm rates and central pressures due to the importance of these parameters and their tendency to vary spatially, though the increased level of effort should be weighed against the advantages of the method. Any new methodologies should be discussed with the Project Monitor.

2.1.5. GEOGRAPHICAL VARIATION OF STORM STATISTICS

In principle, one would expect that the statistical characteristics of hurricanes in the study region would vary as a function of location to some degree. In the Gulf of Mexico, these variations may be due to variations in location relative to the Yucatan and Florida straits and to the loop current. Along the Atlantic coast, these variations may be due to variations in latitude and associated variations in water temperature, prevailing winds, etc.

In many situations, however, the available hurricane data may not be sufficient for resolving these variations, even when there are physical arguments that suggest their existence. In these situations, it is more realistic and sufficient to consider only one distribution of storm parameters over the entire study region.

In the Chouinard approach, the kernel size parameter provides direct information regarding the smallest scale of geographical variation that can be resolved with the available data, with an optimal tradeoff between geographical resolution and statistical precision. In the capture zone approach, one may need to perform additional calculations to determine if there is significant geographical variation in parameters. For instance, one may divide the capture zone and use standard statistical tests to determine whether the capture zone can be treated as having a unique distribution for each parameter. One may also test whether the sample distribution of distance to a suitably defined reference point is consistent with the assumption of a uniform spatial distribution. Care should be taken to ensure spatial variations in parameters that may affect values at study boundaries are accounted for so that results will reasonably match those of neighboring studies.

In some situations, it may be necessary to take data from a broader region, i.e., a region broader than the study region, the capture zone, or the kernel size. For instance, the conditional distribution of R_{max} given ΔP is often determined using data from a much larger region, such as the Gulf of Mexico or the North Atlantic, in order to obtain a sufficient sample; see FEMA, 2008, and Vickery and Wadhera, 2008, for illustrations of this. Still another example of use of an expanded zone to better define a parameter is the model for Holland's B conditional upon Rmax and latitude (Vickery and Wadhera, 2008). In these situations, it is more important that the Mapping Partner obtain a reasonable estimate of the nature of the conditional dependence of the parameter on other hurricane characteristics, than on location.

2.1.6. STORM RATE (SPACE AND TIME) AND THE PROBABBILITY DISTRIBUTIONS OF HEADINGS AND DISTANCE

In the characterization of hurricanes for surge analysis, it is convenient to define the minimum intensity of interest (in terms of a minimum pressure deficit and then develop statistical models for the frequency and characteristics of the storms exceeding that intensity. The choice of this cutoff will influence the range of relative validity of the computed flood statistics. For example, if only storms with low probably of occurrence (such as those affecting the 1% and 0.2% annual-chance events) are of interest, it may be possible to truncate the storm sample so as to include only stronger storms. By not considering weaker storms, the estimates of higher probability events (such as the 10% annual-chance event) may be unrealistic. In order to capture more frequent events, weaker storms would need to be included in the tropical storm range. Choice in target frequencies and frequency cutoffs should be discussed with the FEMA Project Monitor.

This section considers the overall rate of storms, or storm density, in both space and time; subsequent sections then consider the several hurricane characteristics.

It is generally assumed that hurricane occurrences in time are a Poisson process (Parzen, 1962), although data indicates that this is not strictly true. More importantly for practical applications, hurricanes that generate surge in excess of a certain high value of interest (say, 10 feet) at a particular location are assumed to be a Poisson process. The only parameter in this Poisson model is the rate of storms, which has units of storms per unit distance per unit time (e.g., storms/km/year).

If heading is considered as part of the rate calculations, then the rate has units of storms per unit distance per unit angle per unit time (e.g., storms/km/direction-degree/year).

As in many situations involving the study of rare events, the Poisson assumption is actually not necessary for the calculation of rare surge exceedances at a given location, and the results obtained using the Poisson assumption are generally not invalidated by deviations of hurricanes from the assumption of independence of storm rate from other parameters implied by the Poisson assumption.

In the Chouinard approach, the rate at the point of interest is proportional to a weighted count of the observed data in the storm catalog, with weights that depend on the distance from the storm to the site and its deviation from the direction of interest. Storms that pass farther from the site of interest or that have directions different from the direction of interest receive lower weight. The resulting expressions for the directional and omni-directional rates, respectively, are as follows:

$$\lambda(\theta) = \frac{1}{T} \sum_{\substack{i \\ (all\ storms)}} w(d_i) k(\theta_i - \theta)$$

(3)

$$\lambda = \frac{1}{T} \sum_{\substack{i \\ (all\ storms)}} w(d_i)$$

where the summation extends over all storms in the catalog, T is the duration of the catalog (in years), and the kernel weight functions are taken as normal-distribution shapes, as follows:

$$w(d_i) = \frac{1}{\sqrt{2\pi} h_d} exp\left[-\frac{1}{2}\left(\frac{d_i}{h_d}\right)^2\right]$$

and (4)

$$w(\theta_i - \theta) = \frac{1}{\sqrt{2\pi} h_\theta} exp\left[-\frac{1}{2}\left(\frac{\theta_i - \theta}{h_\theta}\right)^2\right]$$

Chouinard and Liu (1997a) introduced a powerful technique to determine the optimal kernel sizes for the calculation of rates, namely least-squares cross-validation. They also consider a related technique, maximum cross-validated likelihood, but the former is preferred because it is more robust. Maximum cross-validated likelihood was used to determine the optimum kernel size for the post-Katrina study (FEMA, 2008), where it was used to determine the optimal kernel-size for the distribution of *ΔP*.

To calculate the optimal kernel width h_d for the omni-directional rate, the data are partitioned at random into two samples (the estimation sample and the validation sample) using a randomization scheme in which each storm is assigned to the estimation sample with probability p and to the validation sample with probability *1-p*. The estimation sample is used to estimate the predicted rate using Equations 3 and 4. The validation sample is then used to calculate the observed rate. The two rates are then adjusted for the size of the two samples (i.e., for the effect of p), and the difference between the two rates is squared. The random partitioning of the sample is repeated many times (say, 500 to 1000 times) and the squared difference is summed over all these random partitions. The resulting quantity is the cross-validated square error (CVSE); the optimal choice of kernel width h_d is the one that yields the lowest CVSE. For the post-Katrina Mississippi study, the observed rate was calculated by counting the number of storms in the validation sample falling within 40 km of the site and then dividing that count by 80 km and by the number of years in the storm catalog. The probability p was set to 0.9 to avoid a large change to the size of the estimation sample. The resulting optimal kernel size is not sensitive to these choices, as long as they are within reasonable bounds (Chouinard and Liu, 1997a). Similarly, the results for directional rates are not sensitive to the choice of angular interval.

In the Capture Zone approach, hurricanes are counted if they cross the coastline (or an idealized representation of the coastline) within the capture zone. The resulting count is divided by the size of the hurricane catalog and by the length of coastline, obtaining a rate of hurricanes per unit length per unit time. The distribution of heading is then estimated based on the empirical distribution of headings observed at landfall. As indicated earlier, this distribution of headings depends on the geometry of the coastline and cannot be compared directly with the distribution obtained using point-based counting. If it is suspected that the rate is not constant within the capture zone, the distribution of distance to some suitable reference point is computed based on the associated empirical distribution.

If the storm rate is truly constant within the study region, then the distance to any conveniently defined reference point (e.g., the mid-point of the region of interest) is drawn from a uniform distribution. This is the most common situation in practice, and will usually be assumed by the Mapping Partner, but it is not always the case. If point-based counting indicates significant variations in rate within the study region, or if the line-based counting indicates significantly different rates for sub-divisions of the capture zone or a distribution of distance that deviates from uniform, then this deviation from uniformly distributed distances must be taken into account. The JPM-OS techniques described in the next section may be easily adapted to include this non-uniform distribution of distance.

2.1.7. STORM INTENSITY

In probabilistic surge studies, the intensity of the storm is characterized by the pressure deficit ΔP, which is defined as the difference between the far-field atmospheric pressure and the central pressure of the storm. The far-field pressure is commonly taken as a fixed value (usually 1013 mb) if better data are not available. Consequently, pressure deficits can be estimated from central pressures reported in HURDAT and elsewhere, by simply subtracting from 1013 mb if data on far-field pressure is not available, though it has become common practice to use the true far-field pressure whenever possible.

The lower-bound *ΔP* of the data used in this step should be consistent with the minimum *ΔP* used for the definition of the rate. That is, the storm rate must correspond to the rate of storms with intensities exceeding the cutoff *ΔP*. In addition, if the statistical distribution shape used includes a lower-bound parameter, this parameter should be selected in a consistent manner.

The distribution shape used for *ΔP* should be consistent with the observed empirical distribution. The most common distribution shapes in recent studies are the Type-I Extreme-Value distribution (also known as Fisher-Tippett or Gumbel) and the three-parameter (or truncated) Weibull distribution (e.g., Resio, 2007, FEMA, 2008, Vickery and Blanton, 2008). It is recommended that one of these distributions be adopted by the Mapping Partner, in accordance with the apparent quality of fit with the study data. Nevertheless, the Mapping Partner may choose another distribution type if the data shows that an improvement would be achieved.

The complementary cumulative distribution function (CCDF) of the Type-I distribution is given by the equation

$$P[\Delta P > x] = 1 - exp\left[-e^{-\alpha(x-U)}\right] \qquad (5)$$

where *U* is the mode of the distribution and *1/α* is a parameter that measures the scale of the distribution; both of these have units of pressure. Note that the Type-I distribution is defined for all real values of *ΔP* (not just for values above the lower-bound used for the calculation of rate. Strictly speaking, the CCDF should be normalized so that $P\,[\Delta P > \Delta P_0]=1$, where ΔP_0 is the lower-bound value of *ΔP* used in the calculation of storm rate.

Similarly, the CCDF of the three-parameter Weibull distribution is given by

$$P[\Delta P > x] = exp\left[-(x/u)^k + (\Delta P_0/u)^k\right] \quad x > \Delta P_0 \qquad (6)$$

where *u* is a scale parameter, *k* is a shape parameter, and ΔP_0 is the lower-bound value of *ΔP* introduced above; *u* has units of pressure, while *k* is dimensionless.

For annual exceedance frequencies of 0.2% or greater (that is, more frequent), the dominant storms tend not to fall too far in the upper tails of the distributions; instead, rarity is more the result of combined moderate parameter values and randomly close proximity, rather than of an extreme value for any one parameter (example: despite its severity, Katrina was a Category 3 storm at landfall). Therefore, the choice of distribution shape used for *ΔP* is likely to have only a moderate effect on the results, whatever form is chosen.

In the Chouinard approach for estimation of the distribution of *ΔP*, the distribution parameters *u* and *k* are estimated from all the storm data using the method of maximum weighted likelihood, where the weights depend on the distance between the track of storm *i* and the point under consideration, and possibly subject to the monotonicity constraint described earlier. Specifically, the weighted log-likelihood is of the form

$$\ln(WL) = \sum_i w(d_i) \ln\left[f_{\Delta P}\left(\Delta p_i; u, k\right)\right] \tag{7}$$

where d_i is the distance between the point under consideration and the linearized track of storm *i* (associated with pressure deficit Δp_i at landfall), $w(d_l)$ is a Gaussian distance-dependent weight (which is given by Equation (4) introduced earlier, although the kernel size h_d need not necessarily the same as for the calculation of rate), and $f_{\Delta P}$ $(\Delta p;u,k)$ is the Weibull probability density function (obtained by differentiating the cumulative function shown above); the summation extends over all storms with ΔP exceeding the lower cutoff of the data set.

Following Chouinard et al. (1997a, 1997b), a technique known as maximum cross-validated likelihood is utilized to determine the optimal kernel size h_d for the estimation of the Weibull parameters. As was done for the calculation of the cross-validated squared error for rates, the data are partitioned into two samples (the estimation sample and the validation sample) using a randomization scheme. The estimation sample is used to estimate the Weibull parameters *u* and *k* by determining the values of *u* and *k* that maximize the log-likelihood function in Equation 6, possibly subject to a monotonicity constraint. The validation sample is then used to calculate the observed log-likelihood. These observed log-likelihoods are then summed over all random partitions of the sample. The resulting quantity is the cross-validated likelihood (CVL; the optimal choice of kernel width h_d is the one that yields the highest CVL).

This analysis may yield an optimal distance-kernel size that is smaller than the optimal kernel size obtained for the directional rates, but the slope in the upper portion of the CVL vs. kernel size has been found to be nearly flat. This result indicates that the cross validation provides only a weak upper bound for the kernel size. In the Mississippi study, the optimal kernel size that was obtained for the directional rates was used for all calculations involving kernels, and it is recommended that the Mapping Partner make a similar assumption unless there is an apparent need for a more detailed analysis for other parameters. Such more detailed analyses are sometimes needed for parameters that exhibit their own sensitivities, such as storm heading and rate. Several recent FEMA studies along the Florida Peninsula, for instance, relied on several different kernel sizes (FEMA 2013, FEMA 2014b).

Once the optimal kernel size is selected, the best-estimate values of the Weibull parameters *u* and *k* are obtained by maximizing Equation 6, possibly subject to a monotonicity constraint.

In the line-based approach, a suitable distribution shape is chosen to fit the empirical distribution of ΔP, using standard statistical methods (e.g., method of moments, maximum likelihood, linear regression on the transformed data). It is also important to investigate geographical variation in the distribution parameters, although the data limitations often yield no statistically significant differences.

In both the Chouinard and Capture Zone approaches, the estimated parameters have a high statistical uncertainty as a result of limitations in the data. In these situations, the exchangeability axiom of modern decision theory suggests that one should use the mean or "predictive" CCDF of ΔP (i.e., the expected value of the CCDF, averaged over the joint distribution of the distribution parameters), not the best-estimate value obtained above. The reader is referred to McGuire et al. (2005) for an elaboration of this issue in the context of earthquakes. Experience with the three-parameter Weibull distribution indicates that there is a significant difference between the mean and best-estimate CCDF because the CCDF is a highly nonlinear function of the distribution parameters, and that the mean CCDF is significantly higher (e.g., Risk Engineering, 2008). In the post-Katrina Mississippi study (FEMA, 2008), a re-sampling technique known as bootstrapping (Efron and Tibshirani, 1993) was employed for the calculation of the mean or predictive CCDF. This approach is very general and is easy to implement. Other approaches, such as standard methods for the propagation of uncertainty may also be employed.

2.1.8. STORM TRACK: FORWARD SPEED OF TRANSLATION

The forward speed of the storm affects the wind field, making it more asymmetrical. It has an additional effect on surge (beyond the effect on wind speeds), in that it helps determine duration of high water (and so, perhaps, overtopping and filling volumes). There are physical arguments that suggest a positive correlation between forward speed and ΔP, but the available data generally show a weak or non-existent correlation. Storms making landfall in the Atlantic seaboard tend to have somewhat higher forward speed than those in the Gulf of Mexico as a result of differences in the steering winds.

The associated probability distributions can be taken as normal or log-normal by the Mapping Partner, based on examination of the associated empirical distribution. Given the associated coefficients of variation and the moderate importance of forward speed in the calculations, the practical effect of choosing a different distribution is anticipated to be small.

2.1.9. STORM SIZE

The radial dimension of the tropical storm wind field has a large effect on surge, as demonstrated by Irish et al. (2008). Recent studies have utilized a single parameter to characterize this size, although there is a trend toward allowing this parameter to vary as a function of quadrant.

Two parameters are commonly used to represent storm size, namely the radius of maximum winds, R_{max}, and the characteristic radius R_p of the exponential pressure profile, where the pressure profile is written as

$$p(r) = P_0 + \Delta P \exp\left[-\left(\frac{R_p}{r}\right)^B\right] \tag{8}$$

in which P_0 is the central pressure and B is Holland's shape parameter (Holland, 1980), to be discussed below; B is often taken as unity, although it can have a notable influence on winds and surge. There is only a slight difference between these two radius measures for typical cases, and some studies have ignored the difference, or failed to recognize it. The difference may be large for the profiles of real hurricanes, however, as these may have quite irregular shapes. Consequently, the Mapping Partner should take care to distinguish between them in collection and analysis of the study data.

Most studies find a weak negative correlation between R_{max} (or R_p) and ΔP. In the post-Katrina Mississippi study, an expression of the form

$$\ln[R_{p,median}] = 4.37 - 0.29\ \ln[\Delta P] \tag{9}$$

was obtained using linear least squares regression. Similarly, Vickery and Wadhera (2008) obtained relations of the form

$$\ln[R_{p,median}] = a - b\Delta P^2 + c\,\psi \tag{10}$$

for Atlantic and Gulf of Mexico hurricanes, where ψ is latitude; the parameters *a* and *b* vary with region. Other recent FEMA studies have developed their own relationships based on historical data. The Mapping Partner shall make a similar determination for the study data, as appropriate. Most studies model the conditional distribution of R_p given ΔP as lognormal, the associated standard deviation of $\sigma_{\ln[Rp|\Delta P]}$ is generally found to be approximately 0.4 to 0.5. Vickery and Wadhera (2008) find that stronger hurricanes exhibit a lower standard deviation and provide equations for $\sigma_{\ln[Rp|\Delta P]}$ as a function of ΔP.

Although correlation between R_p and ΔP is weak — or apparently non-existent for certain subsets of the data such as Gulf of Mexico hurricanes at landfall (Vickery and Wadhera, 2008) —most studies assume a negative correlation. Storm physics modeling also provides support for a negative correlation (Shen, 2006). Consequently, it is recommended that the Mapping Partner should assume a correlation, although some effort may be required to assemble the necessary data and it may prove necessary to enlarge the capture zone so as to make the correlation apparent. Note that this matter is discussed at some length in NWS-38 (Ho, et al., 1987) which may also be consulted for guidance.

2.1.10. OTHER PHYSICAL PARAMETERS

The wind fields of real hurricanes vary widely with both distance and azimuth from the storm center, and cannot be completely characterized by only two parameters, namely ΔP and R_{max}. The natural choice for the next major parameter to include in a JPM analysis is Holland's *B* parameter, which was introduced in Equation 8 above. Higher values of *B* produce more highly peaked wind fields, with higher values of the peak wind speed. On the other hand, this peak value occurs over a narrower spatial reach. As a result of these counteracting effects, the sensitivity of peak surge and surge hazard to *B* is not clear at present. According to Irish et al., (2009), the effect of changing *B* from 0.9 to 1.9 is to give a change of the order of 15% in peak surge. It is recommended that this matter be given attention by the Mapping Partner through numerical experiments with the hydrodynamic model and alternate choices of *B*. There have been few statistical studies for *B*. Vickery and Wadhera

(2008) find a weak correlation with ΔP, R_{max}, latitude, and sea-surface temperature. Unless a preliminary sensitivity investigation suggests otherwise, the Mapping Partner may adopt a simplified representation of *B* for the JPM simulations, such as (1) using a single mean value of *B*, (2) allowing *B* to vary probabilistically in the synthetic storm population, or (3) via the post-computation error approach to be discussed later. If *B* is found to be important, then it may need to be represented as a JPM parameter. However, the relative cost of this, versus representing other parameters in greater detail, such as tides and model resolution, should be weighed carefully.

Additional parameters have been used to describe hurricanes, for instance, Cox and Cardone (2007) use a double-exponential model to describe the hurricane wind field in hindcast studies. This model has the form

$$p(r) = P_0 + (\Delta P - \Delta P_2) \exp\left[-\left(\frac{R_{p,1}}{r}\right)^{B1}\right] + \Delta P_2 \exp\left[-\left(\frac{R_{p,2}}{r}\right)^{B_2}\right] \tag{11}$$

Storms that are better represented by a double exponential model tend to have a broader radial wind and pressure profile, such as Hurricanes Dennis (2005), Katrina (2005), and Ike (2008). This tends to enhance storm surge, especially beyond the radius of maximum wind. This model introduces three additional parameters, whose statistics (potentially including correlations with other parameters) must be determined. Currently, most historical hurricane data sets do not include these additional parameters, making the necessary calculations impossible, but there are re-analysis efforts under way that will determine these values in the future. However, the burden of expanding the full JPM to include 3 additional parameters could be significant, making it unclear at what point or under what circumstances the expansion of the JPM space will be worthwhile. Approximate methods to account for the effect of this distribution may also be performed, however this field has not yet been well-explored. Three recent FEMA studies, covering all the western coast of the Florida peninsula, included the bulk effect of this broader wind field through a customized, spatially variable uncertainty model (FEMA 2013, FEMA 2014a, FEMA 2014b, FEMA 2016a). This was done due to evidence that these parameters had an important role in storm surge generation in the area. Future studies may attempt to include these additional parameters or may follow the path taken by the post-Katrina FEMA study for Mississippi to use a simpler representation of these parameters. Like Holland's B, however, the relative importance and cost of the additional parameters should be weighed against the benefits to the study.

2.1.11. TREATMENT OF PARAMETER CORRELATIONS

In principle, all hurricane characteristics are correlated to some degree. Most probabilistic surge studies, however, consider only the correlation between ΔP and R_{max}. The main reason for modeling the pressure-radius correlation--despite statistical r^2 values on the order of only 0.3—is that these are the two most important hurricane parameters for surge calculations (other than landfall location, taken independent by assumption). Allowing for the inverse pressure-radius correlation ensures that extremely intense storms are not assigned extremely large radii in construction of the JPM storm simulation set. The important major correlation of parameters with track angle has been commonly treated by the simple artifice of dividing the storm sample into entering and exiting populations; within each subpopulation, independence with angle is assumed.

2.2. Storm Simulation Set – JPM-OS Method

At least four distinctly different JPM approaches have been used for FEMA flood insurance studies. As noted earlier, the original work by Ho and Myers (Ho and Myers, 1975; Myers, 1975) adopted a direct JPM approach based on the idea of dividing each parameter distribution into a small number of segments, and then simulating all possible combinations (all possible storms). As noted earlier, the only difficulty with this approach is that the number of simulations that are required quickly becomes prohibitive, especially when considered in light of the computational demands made by modern high-resolution hydrodynamic models such as ADCIRC.

In the post-Katrina efforts of the USACE (for Louisiana) and FEMA (for Mississippi) greatly improved JPM methods were developed, permitting a JPM analysis to be performed with only about one tenth the number of simulations as would be required with the direct JPM approach. These are now known as Optimal Sampling methods. These guidelines correspond to the approach used in the FEMA Mississippi study, called the Quadrature Method, although the USACE's approach for Louisiana, called the Response Surface Method, is also entirely appropriate for new FEMA studies. Experience gained during the post-Katrina work showed that the two approaches are capable of giving nearly identical results with a similar level of effort. These guidelines focus on the Quadrature Method since it is more readily automated than the Response Surface Method which requires a greater degree of expert judgment in the selection of storms. Recently, a third approach has been followed in a study of North Carolina (see Vickery and Blanton, 2008). It is not described in detail here, but it is noted that it is not, strictly speaking, an optimal sampling approach. It is more akin to a traditional JPM approach in that the parameter distributions are discretized and all combinations are simulated. Furthermore, it departs from the other approaches in that number and locations of the tracks are not established in a defined pattern but are distributed over the coastal area using Monte Carlo simulation, with the assumption of a Poisson occurrence rate. Whereas the Quadrature and Response Surface methods have been compared and found consistent, such a comparison with the North Carolina approach has not yet been made.

2.2.1. SUMMARY OF THE RESPONSE SURFACE METHOD

This approach takes advantage of the following three observations from sensitivity studies performed during the post-Katrina studies: (1) the calculated surge η_m is a fairly smooth function of the storm parameters; (2) η_m is most sensitive to ΔP, R_p, and track location (or along-coast distance between storm track and location of interest); and (3) the sensitivity of η_m to heading angle θ and forward velocity V_f is weaker and may be approximated as linear. Furthermore, the variation of η_m as a function of these parameters is fairly smooth. These observations are confirmed by the sensitivity analyses documented in FEMA (2008) and by model runs cited by Resio (2007).

As a result of these observations, it is possible to perform surge calculations for a moderate number of synthetic storms—with carefully selected combinations of parameters--and then to interpolate between the calculated surge elevation (in the JPM dimensions) to obtain the surge elevation for any desired combination of parameters. The computational cost for this interpolation is minimal. As a result, one can discretize the domain of the JPM integral very finely, even in five dimensions.

The main difficulty in the response-surface JPM-OS scheme resides in the experimental design (i.e., the selection of the parameter combinations for the synthetic storms) in a manner that provides enough points in the highly dimensional JPM parameter space, without requiring a very large number of synthetic storms, and then implementing a robust interpolation scheme that works reliably for all target sites of interest. This selection process and interpolation scheme treats ΔP and R_p as the primary variables for each selected track and takes advantage of the weak sensitivity to heading angle θ and forward velocity V_f, which are treated as linear. The interpolation between tracks is more delicate and requires special treatment, as described by Irish et al. (2009).

The application of the Response Surface Method in Louisiana and Texas is summarized in Resio (2007), Resio et al. (2009), and Irish et al. (2009). These publications provide details on the application of the method and show typical results. In particular, it has been found that the approach yields results that are essentially indistinguishable from the Quadrature Method (Toro, et al, 2010).

2.2.2. SUMMARY OF THE QUADRATURE METHOD

Gaussian quadrature is a well-known technique for approximation of integrals of the form

$$I = \int f(x)\ p(x)dx \qquad (12)$$

where $f(x)$ is often a probability density function (i.e., it is positive and it integrates to unity) and $p(x)$ is a function belonging to a particular family of functions. The quadrature approximates the integral as a finite weighted summation of the form $I \approx \sum I\ w_i\ p(x_i)$, where the nodes x_i and the corresponding weights w_i are selected in a manner that maximizes the accuracy of the approximation, while keeping the number of nodes small. In the context of the JPM-OS Quadrature method, each node can be thought of as one synthetic storm. The weight associated with each node is multiplied by the annual rate of storms to obtain the annual rate for that synthetic storm.

In one-dimensional Gaussian Quadrature, the number of nodes, the nodal locations, and the weights are selected so that the summation will evaluate the integral exactly if $p(x)$ is a polynomial of a certain degree and $f(x)$ is a particular probability distribution (e.g., a standard normal probability density). This technique is used frequently in one dimension. Miller and Rice (1983) provide implementation details and results for a variety of commonly used probability distributions. The improvement in calculation efficiency (number of function evaluations needed for a specified accuracy) can be very great compared to simpler methods.

It is also possible to fix the weights w_i to arbitrary values (e.g., equal weights or 1/6, 2/3, 1/6) and then calculate the nodal values x_i so that polynomials of a certain degree are integrated exactly (or, equivalently, so that distribution moments up to a certain order are preserved by the x_i, w_i pairs). In addition, it is possible to fix the nodal values and then compute the required weights.

Unfortunately, extension of these so-called zero-error one-dimensional rules to more than one dimension is problematic. It is easy to apply a one-dimensional quadrature for each parameter and then generate all possible multi-dimensional parameter combinations. These so-called product rules result in a large number of nodes. Furthermore, if Gaussian Quadrature is used, many of these combinations will have very low weights. The more efficient techniques to generate multi-dimensional Gaussian Quadratures often lead to some weights being negative, which create stability problems and make it impossible to interpret the weights in terms of the occurrence rates of synthetic storms.

The product rules mentioned above have some practical applications. In particular, one can use a product rule to construct a JPM Reference Case which is then used -- together with a fast surge code such as SLOSH or a low resolution ADCIRC model to validate a more efficient multi-dimensional Quadrature. In addition, a product rule constructed from 3- and 4-point quadratures with equal weights is being used in the recent surge study for North Carolina (Vickery and Blanton, 2008). In contrast to Gaussian Quadrature, Bayesian Quadrature (also termed Gaussian-Process quadrature) defines the family of functions *p(x)* as all possible realizations of a random process having a certain auto-covariance function and seeks to minimize the integration error in a mean-squared sense instead of trying to make it equal to zero. The main advantage of the probabilistic formulation of the quadrature problem is that the formulation is easy to apply in multiple dimensions. In addition, it is possible to control the accuracy of integration in each dimension by adjusting the parameters of the auto-covariance function.

The Quadrature JPM-OS approach, as applied to date, uses Bayesian Quadrature in conjunction with more traditional numerical-integration schemes to transform the JPM integral into a discrete summation with a moderate number of nodes. The result is a set of synthetic storms, where each synthetic storm is defined by its parameters at landfall (i.e., ΔP, R_p, V_f, track location, etc.), and each synthetic storm has an associated annual recurrence rate. Typically, a few hundred synthetic storms (rather than a few thousand) are sufficient to attain the desired accuracy. For numerical reasons, the calculation of the optimal nodal locations and associated weights is performed in standard multi-dimensional normal distribution space. The nodal locations are then mapped into the physical space of ΔP, R_p, V_f, etc.

2.2.3. QUADRATURE METHOD OF STORM SELECTION

This section presents the recommended Quadrature method, based upon its use in the post-Katrina Mississippi study. It is anticipated that the method will evolve as it is exercised in future studies.

The quadrature approach to define a representative set of synthetic storms and their associated annual rates and uses a combination of traditional and sophisticated numerical-integration schemes. The process may be summarized in the following fundamental steps:

- Discretize the distribution of ΔP into broad slices, roughly corresponding to Saffir-Simpson hurricane Categories and compute the probability mass contained in each slice.

- Within each ΔP slice, discretize the joint probability distribution of ΔP (within slice), R_p, V_f, θ, and any other JPM variables using Bayesian Quadrature. Details on this step are provided subsequently.
- Discretize the distribution of landfall location by replicating each of the synthetic storms defined in the previous two steps at spatial offsets equal to R_p (measured perpendicular to the storm track). To avoid aliasing, apply a random perpendicular offset (with a uniform distribution between 0 and R_p) to each replicated set of storms. Sensitivity studies indicate that a spacing of Rp is small enough to capture the peak surge at all grid locations.
- Compute the probability p_i assigned to each synthetic storm as the product of the probabilities resulting from the previous three steps. Then, compute the rate l_i assigned to each synthetic storm as the probability p_i computed above times the rate per unit length times the storm spacing.

These steps are discussed in more detail in what follows.

2.2.3.1 Inputs

The first set of inputs to the Bayesian Quadrature algorithm consists of the probability-distribution information for the hurricane characteristics at landfall, namely ΔP (within slice), R_p, V_f, and θ, and possibly other characteristics. For each hurricane characteristic, this information consists of the distribution shape (e. g., Weibull, Gumbel, lognormal) and distribution parameters. For dependent hurricane characteristics such as R_p, these distribution parameters are functions of ΔP.

The second set of inputs consists of information on the characteristics of the surge response. Because of the probabilistic nature of the Bayesian Quadrature method, this information is of a probabilistic nature, and consists of the correlation distances of the term

$$P\,[h_m\,(\Delta P,\, R_p,\, \text{etc.}) > h] \qquad (13)$$

in Equation 13 along the various dimensions. The higher the sensitivity of the quantity $P\,[h_m\,(\Delta P, R_p, V_f, \theta, \text{etc.}) > h]$ to a particular hurricane characteristic, the lower the corresponding correlation distance. These correlation distances are not specified in the physical units of the hurricane characteristics. Instead, they are specified in the corresponding normal-distribution space used internally by the Bayesian Quadrature algorithm. Estimates of these correlation distances could be obtained from sensitivity results, such as those generated for the Mississippi study, but have been specified on the basis of judgment. The following values are suggested for guidance for the choice of correlation distances in a new FEMA study:

- Sensitive (important): Pressure and Radius: correlation distances of 1 to 3
- Insensitive (less important): Forward speed, direction: correlation distances of 4 to 6

In a relative sense, the Quadrature JPM-OS algorithm tends to spread the sampling nodes more widely along those directions with lower correlation distances, providing a closer match to the marginal probability distributions in those directions. Thus, it is important to specify correlation distances that relate to the importance of the various physical quantities, in order to obtain an optimal allocation of effort among the various dimensions.

In an absolute sense, numerical experiments in one dimension show that low values of the correlation distance cause the algorithm to be more cautious and to tend towards equal weights, while high values provide a wide range of weights and sample points that extend farther into the distribution tails, approaching those obtained by Gaussian Quadrature. The ideal choice is somewhere in between.

It is reasonable to assume that relative parameter importance is likely to be similar in a new study to their importance as estimated by detailed sensitivity tests in the post-Katrina Mississippi study. Consequently, the assumptions used there should be reviewed by the Mapping Partner and can be followed unless there is reason the suspect that alternate choices should be used.

An additional input is the number of nodes to generate. In the Mississippi application, different numbers of nodes, and somewhat different correlation distances, were employed for the various ΔP slices. The number of nodes in a slice ranged from five to seven and can be followed as precedent for a new study.

2.2.3.2 Algorithmic Steps

The first algorithmic step employed in the Bayesian Quadrature is the selection of the optimal nodal locations (in normal-distribution space) and the associated weights. This is achieved by using two nested optimizations, both of which seek to minimize the variance of the integration error. At the inner level of nesting, there is the optimization to determine the best weights (for given nodal locations). This is done in closed form, by solving an optimization problem not too different from linear least squares. At the outer level, there is the search for the best set of nodal locations. This is done using a numerical optimization scheme. Details on the formulation and implementation of both optimizations are provided in Toro et al. (2007, 2010).

The second step is the mapping of the nodal locations from standard normal-distribution space to the physical space of ΔP, R_p, V_f, etc. This is done by using the so-called Rosenblatt transformation (see, for example, Madsen et al., 1986; Melchers, 1999). In one dimension, this transformation simply maps each normally-distributed nodal value by finding the value of the physical quantity that has the same value of the cumulative distribution. Extension to multiple dimensions is straightforward, as one can usually write the joint cumulative distribution of the hurricane characteristics as a product of marginal and conditional distributions, e.g., $F_{\Delta P}\ F_{Rp|\Delta P}\ F_{Vf}$, etc., allowing the sequential application of the one-dimensional transformation. The Rosenblatt transformation allows practical implementation of Bayesian Quadrature for virtually any choice of joint probability distributions, as required for JPM-OS.

2.2.3.3 Verification of Storm Selection Step

Because the Quadrature JPM-OS formulation involves some simplifying assumptions regarding the properties of the auto-covariance functions, and because the parameters of this function are chosen on the basis of judgment, it is recommended that the accuracy of the synthetic storm set be validated. This may be done by creating a larger (typically a few thousand) reference set of synthetic storms using a conventional JPM formulation, calculating surge for both sets using a fast hydrodynamic program such as NOAA's SLOSH model, and comparing the resulting flood hazard.

The verification performed in the Mississippi study (FEMA 2008) provides good guidance in this regard and should be studied by the Mapping Partner. The following are some of the key features of this exercise. The probability distributions for the Reference JPM scheme were discretized using one-dimensional Gaussian Quadrature and then all combinations were generated (i.e., a product rule was used). The number of points in these quadratures varied as a function of importance, using 6 nodes for ΔP, 5 nodes for $R_p|\Delta P$, 3 nodes each for forward speed and for heading, and a track spacing equal to R_p. Surge calculations for the JPM-OS scheme and the Reference scheme were performed and compared for a large number of grid points distributed throughout the study region; comparisons were performed for the surges associated with both the 1% and 0.2% annual exceedance chances. Whereas the Reference scheme involved several thousand storm simulations, satisfactory OS schemes of less than 200 storms were identified, showing deviations from the Reference results of better than 1 foot of surge.

It may be possible to streamline this verification by reducing the number of grid points considered or by using a parametric surge model (e.g., Irish et al., 2008). The reduction in the number of grid points brings only moderate savings. The use of the parametric source model brings significant savings but may only be appropriate for uncomplicated coastlines.

There are other simple procedures to verify the adequacy of the JPM-OS storm selection and rates, as follows:

- Comparison of statistical moments of the original (continuous) distributions to those calculated from the JPM-OS discretization. As a minimum, the marginal moments up to order three and the covariance between ΔP and R_p should be checked.
- Graphical examination of the cumulative distribution of calculated surge obtained at several grid points. Ideally, this distribution should have no large jumps in the regions of interest (the region between 10% and 0.2% annual exceedance chances). Large jumps indicate that the hazard is controlled by one (or a few) synthetic storms, suggesting that the JPM-OS storm set needs to be refined.

Given the limited practical experience with the JPM-OS discretization, these simpler procedures would not constitute a replacement for a SLOSH-based or parametric-model based verification of the selected JPM-OS storm set.

In past studies, these verification exercises have been performed prior to introducing the contributions of the small random error terms in the calculated surge (to be discussed below). This is conservative, in the sense that the JPM-OS procedure is likely to be more accurate than the verification tests indicate. The effect of integration over the small error terms is to make $P\ [h_m\ (\Delta P, R_p, V_f, \theta, \text{etc.}) > h]$ a smoother function of the hurricane characteristics, making it easier to integrate numerically.

2.2.4. DEVELOPMENT OF A COMPLETE STORM HISTORY

Both the Response-Surface and Quadrature JPM-OS approaches characterize each synthetic storm by means of the values of the storm's characteristics (i.e., ΔP, R_p, V_f, θ, landfall location, etc.) at landfall (or at some arbitrary location prior to landfall). The numerical ocean-response models require a complete history of hurricane characteristics and eye coordinates for a period of several days prior to landfall.

In recent studies, the storm characteristics prior to landfall have been treated as deterministic functions of the characteristics at landfall. These functions have included some weakening immediately prior to landfall. In the models used recently for the central Gulf of Mexico, all but the storms with very small radius begin to weaken, increase their radius, and decrease their Holland's B over the last 90 miles prior to landfall (Resio et al., 2009). Similar models have been developed for storms affecting North Carolina (see Vickery and Blanton, 2008). In principle, these variations in storm characteristics should also be treated as random, but this is difficult to do within the present JPM-OS formulation, without unrealistically enlarging the dimensionality of the problem (beyond the adequacy of the data).

It is also important to use realistic track geometries, mostly for the purpose of calculating the waves that tend to accompany the surge and which, in fact, contribute to the surge through the intermediate mechanism of the wave's radiation stresses. In the central Gulf of Mexico, examination of the tracks from strong storms indicates that they tend to enter the Gulf through the Florida or Yucatan straits and then follow simple tracks, which may be easily mimicked using simple deterministic algorithms. These algorithms generate a track for any given landfall location and heading. Specifically, a historical hurricane's track can be used as the base offshore path, and then the exact track can be constrained to the desired landfall location and heading by modeling the track as a damped harmonic oscillator. Solution to such equations is readily achieved by standard methods. A similar approach has been followed for North Carolina (see Vickery and Blanton, 2008), but with models that exhibit significantly less weakening just prior to landfall. Although the tracks are idealized, they are chosen to follow the main trends of the observed track history — the landfall track configurations are, of course, directly determined by the parameter selections at landfall, so idealization of the offshore track segments is acceptable. Note that this treatment is superior to the approach used in early flood insurance studies, which assumed simple fixed straight tracks throughout the duration of a storm.

2.3. Second Order Concerns

2.3.1. SMALL RANDOM CONTRIBUTIONS

The procedures described in Section 2.2 will not always include all factors which contribute to a best estimate of surge height. In order to minimize the number of storms to be simulated, some minor or secondary factors may be treated by an approximate method. Furthermore, random uncertainties associated with modeling errors in both meteorology and hydrodynamics also affect the best estimates.

The relationship given in Equation 2 can be expanded to include these factors by inclusion of the term ε (the probability integral is here shown as the discrete summation over the simulation storm set):

$$P[\eta_{max(1\,yr)} > \eta] \;\approx\; \sum_{i=1}^{n} \lambda_i P[\eta(\underline{x}_i) + \varepsilon > \eta] \tag{14}$$

where ε might consist of several constituents, such as:

ε_1 – representing the astronomical tide level as a random function of time, estimated from a local hurricane season tide prediction, and characterized by a standard deviation around zero mean.

ε_2 – representing variations in surge response caused by random variations of the Holland's B parameter that are not represented in the modeling. The standard variation for this term may be dependent upon the computed surge elevation.

ε_3 – representing random errors in the computed surge caused by lack of skill of the numerical modeling. This can be estimated by comparisons of predictions with high-water marks.

ε_4 – representing variations in the surge due to a wide range of departures in the real behavior of hurricane wind and pressure fields that are not represented by the PBL or other meteorological model used to describe the storms. This can be evaluated by comparing the results of surge modeling done using hand-crafted 'best winds' with the findings for the same storms as represented using the PBL model chosen for the simulations.

These and other components of ε, as necessary, are taken to be independent, and so can be combined into a single term having a standard deviation given (with obvious notation) by:

$$\sigma_\varepsilon = \sqrt{{\sigma_{\varepsilon_1}}^2 + {\sigma_{\varepsilon_2}}^2 + {\sigma_{\varepsilon_3}}^2 + {\sigma_{\varepsilon_4}}^2} \tag{15}$$

For each of these components, and others as may be identified, the Mapping Partner shall estimate the standard deviations. For example, a Mapping Partner could follow the precedent set in the post-Katrina Mississippi study. Note that tide cannot always be treated as a small linear addition (see the following subsection). However, when it can, the necessary sigma is easily estimated from local tide predictions, restricted to hurricane season. One of the effects of introducing these secondary factors is that they smooth out the *P*[ ] term in Equation 13, making numerical evaluation somewhat easier.

2.3.2. ALTERNATE METHODS FOR LARGE AMPLITUDE TIDES

In the event that tide amplitudes are not small compared to the 1% stillwater elevation level (SWEL), or if the Mapping Partner has reason to doubt the validity of representing tides as a secondary factor owing to great distances of inland propagation of storm surge and the like, treating tide as a small additive correction will not be appropriate. No simple method has been identified to handle the tide in such cases. Note, too, that in some cases the tide may be small compared to the 1% SWEL, but not compared to the 10% and 2% SWELs. The relative error in those cases may then be greater, although still smaller in an absolute sense. Whether these other levels must be given the same degree of attention as the 1% and 0.2% SWELs in a particular study, should be determined by the Mapping Partner in consultation with the FEMA Project Monitor.

When representation of tides as a secondary factor is inadequate, the Mapping Partner may adopt any of several alternate methods. Detailed discussion of incorporating tides into surge is presented in *Coastal Water Levels* (FEMA 2016b). These include linear superposition, random tide phase assignment to individual storms, running each storm at several tide phases, or the more complex approach detailed in FEMA's User's Manual for the FEMA Coastal Flooding Storm Surge Model (FEMA, 1988). The approach is discussed in full detail in Chapter 8 of Volume 1 of that document. There are also computer codes (presented in Volume 3) which may be used to help guide new work. In brief, the procedure recommended is to simulate a small number of representative storms not only at mid-tide (as is done for the full storm simulation set), but also at other tide levels and relative phases. These hydrodynamic simulations properly incorporate the interactions of surge and tide throughout the study area and can be compared with estimates based on linear addition. The comparison of these two calculations is then used to define regression expressions that are used to adjust the estimates obtained by linear addition so as to better approximate the full simulations. The approach is relatively time consuming and should be accounted for in the initial study scoping with concurrence of the FEMA Project Officer.

2.4. Surge Frequency Determination

2.4.1. OVERLAND DISTRIBUTION OF TARGET SITES

Surge statistics are required at enough points distributed throughout the study region to permit accurate mapping of flood zones and to permit the prior determination of overland wave crests.

The simplest selection of target points would correspond to the nodes of the hydrodynamic model since surge elevations are computed at each node throughout the simulation of a storm. However, many modern models are typically run with extremely fine resolution, so that several hundreds of thousands of grid points might fall within the area of interest. Such extreme density may not be required for preparation of flood hazard maps. Consequently, the Mapping Partner may select an adequate subset of points for the statistical analyses. This introduces further complications, however, in areas where surges and SWELs change rapidly, as is often the case in coastal areas with barrier islands and other major topographic features, and the Mapping Partner must take care to properly account for these smaller-scale spatial variations. As a practical matter, however, given the availability of large computers and inexpensive data storage, it may be simplest just to include all points for analysis and, in a later step, produce a SWEL surface from which the necessary mapping information can be easily extracted. Usage of all points in the domain has become standard practice in all FEMA studies since 2010.

2.4.2. CONSTRUCTION OF THE SIMULATED DENSITY DISTRIBUTION HISTOGRAMS

Once the JPM-OS storm simulations have been completed, and any necessary adjustments for secondary factors such as large amplitude tide have been accounted for (but exclusive of the small factors treated as random error terms), the final determination of flood frequency at a given point follows using the methods which have been used in past FEMA studies and which are detailed in the FEMA Coastal Flooding Hurricane Storm Surge Model documentation (FEMA, 1988) and in the report of the post-Katrina Mississippi study (FEMA 2008).

Focusing on a single site within the study region, the key idea is to construct a histogram of accumulated rate versus peak surge elevation similar to the example shown in Figure 2. The histogram consists of bins of elevation with suitably small widths (such as 0.1 feet) extending from zero to a bin exceeding the largest surge of interest. Then the rate associated with each of the simulated storms (as determined using the Quadrature method outline above) is accumulated into the particular bin corresponding to the peak surge at the site for that storm. With a fine resolution of bin width, many bins will, of course, remain empty, and the final histogram is an estimate of the surge probability density function.

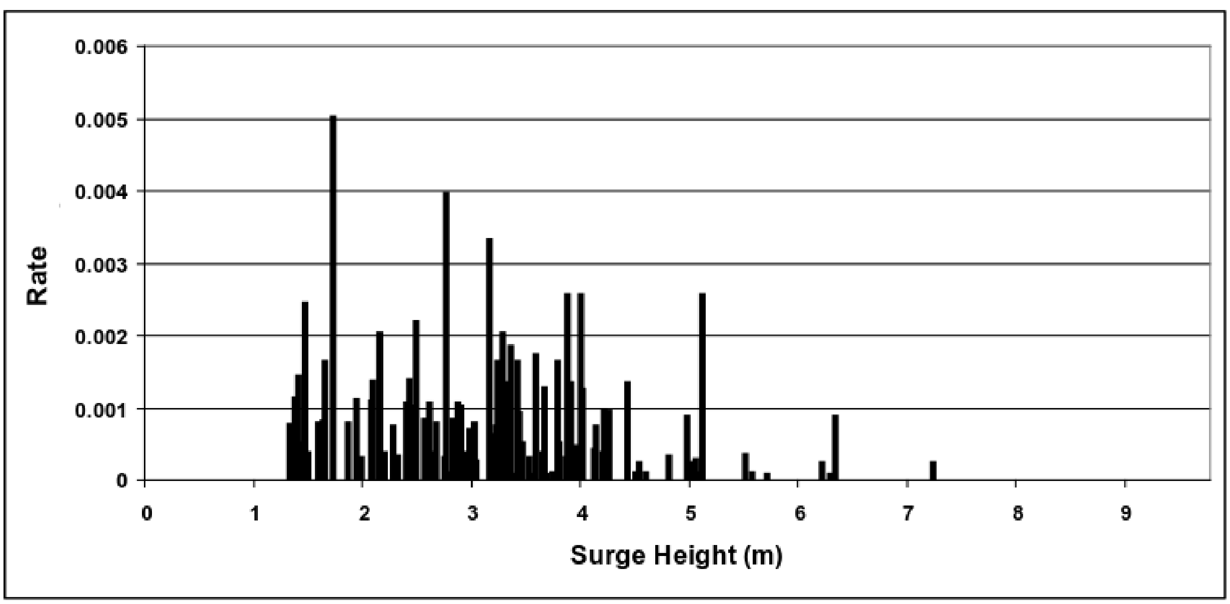

Figure 2: Histogram generated for a single JPM point based on surges and storm rates

Were there no small secondary factors to be accounted for, this estimate of the density distribution is then summed from the top down to produce the corresponding estimate of the cumulative distribution.

The surge elevations at any frequency of interest are obtained from the cumulative distribution, by simply entering the distribution at the specified frequency on the vertical axis and reading across to the curve and down to the nearest bin. The nearest bin will give the corresponding surge elevation to the bin resolution.

More details on this topic can be found in *Coastal Water Levels*.

2.4.3. HISTOGRAM ADJUSTMENT FOR SECONDARY RANDOM FACTORS

In order to account for the secondary (epsilon) terms, one adopts an extremely simple procedure. Consider, as shown in the upper portion of Figure 3 the accumulated rate contained in a single bin of the density histogram. The assumption is that owing to the small random variation associated with the secondary terms, this quantity of rate could be smeared over an interval of elevation bins above and below the original bin. This redistribution is shown in the lower portion of Figure 3, and is simply a discrete approximate to the Gaussian having a width determined by the composite standard deviation given by Equation 14.

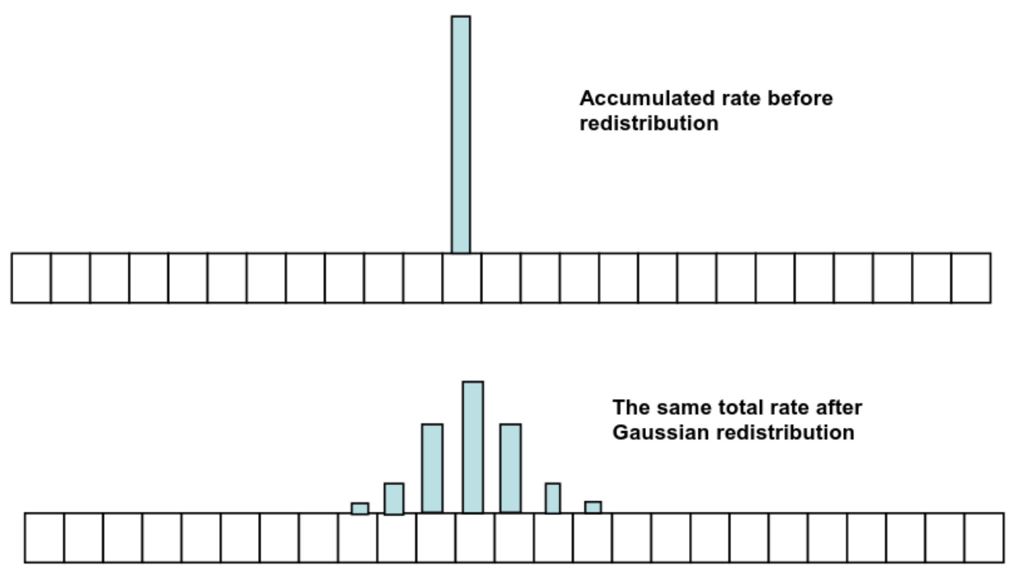

Figure 3: Example of redistribution of the accumulated rate within a single bin to account for secondary random processes

This same sort of redistribution is performed for each bin in the original histogram. Note that in general the contribution of a particular factor may not be constant but may be dependent upon the magnitude of the surge and on the bin location. Once the redistribution of bin rates has been completed, the revised density distribution is summed from the top down, as described before, to yield the cumulative distribution shown in Figure 4, below. Keep in mind that this distribution is unique to a site, so that many thousands of such computations will be needed, depending upon the density of target sites selected for mapping purposes.

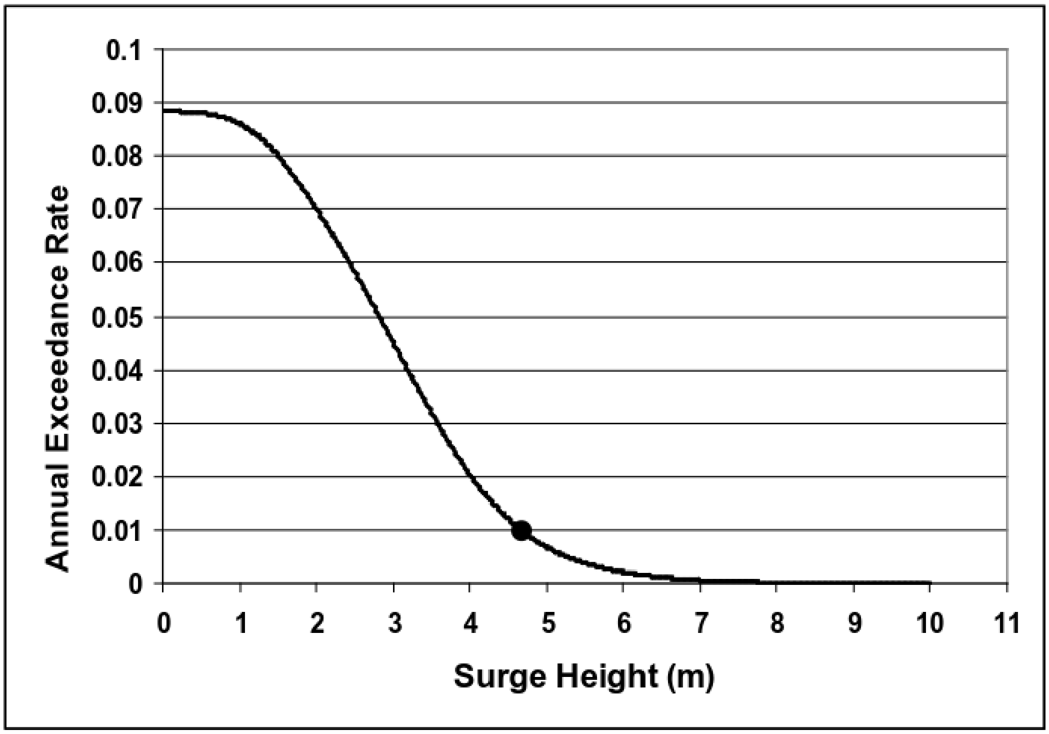

Figure 4: Determination of the 1% stillwater elevation (SWEL) from the top-down integrate rate histogram

3. Distribution Fitting Methods

While the JPM methods discussed in Section 2 are appropriate for coastal flooding associated with hurricanes that can be parameterized with key characteristics, assessment of coastal flooding from other storm sources requires differing methods. The distribution fitting methods are divided into non-parametric and parametric approaches. Parametric methods involve fitting a known distribution such as the Generalized Extreme Value (GEV) and the Generalized Pareto Distribution (GPD) to the data.

The most common non-parametric approach is the Empirical Simulation Technique (EST). The USACE developed the EST for use in storm surge analysis in the 1990s. A full discussion of the method can be found in Scheffner et al. (1996). In the discussion below, the general approach for applying the EST to tropical storms is described first for reference, followed by a description of both the EST and parametric approaches for analysis of extratropical storms.

3.1. Historical Application of the EST for Tropical Storm Analysis

At the core of the technique is the estimation of the cumulative distribution using non-parametric plotting position methods. The implementation begins with identification of the historical dataset of tropical cyclones. From this historical dataset, a subset of storm events referred to as the ‘training set” is selected such that it is representative of the entire set of historical storms. The training set may be augmented with a perturbation to a historical event such as a tropical cyclone with a slightly altered path. For example, since severe hurricanes are rare events, the largest hurricane in the study area may be replicated along different tracks so that its hypothetical impact at other areas in the study area is represented in the analysis. Some historical events may also be excluded from the training set if two events are nearly identical such that both would produce the same response. Each tropical storm identified in the training set is simulated with numerical models to generate the storm’s induced surge peak elevation throughout the study area. Wind fields are generated using a meteorological model and then the wind fields are applied in a circulation and wave model to determine the surges throughout the study area.

After the model simulations area completed, cumulative frequency distributions are developed using the plotting position method with the peak elevations for all locations of interest in the study area. However, prior to completing the plotting position calculations, the dataset is extended beyond the historical period of record to obtain cumulative distributions for low frequency events not included in the historical dataset. The extension is obtained by using the training set and a boot-strapping random walk process. The random walk consists of defining a set of input vectors and associated response vectors. In the case of tropical cyclones, pertinent input vectors include central pressure deficit, radius to maximum winds, maximum winds, minimum distance from the eye of the storm to the location of interest, forward speed of the eye, and tidal phase during the storm event. The response vector is the peak surge associated with the storm. With these definitions, one random walk consists of randomly picking a storm and then taking a random walk in the direction of nearest neighbor events. The walk is based on independent uniform random numbers ranging from -1 to 1 and has the effect of simulating responses that are not identical to the historical events themselves but are similar to those events that have historically occurred.

The random sampling of the finite-length historical-event database generates a larger long-period database, which can be used to assess the importance of variability. The only assumption is that future events will be statistically similar in magnitude and frequency to those particular storms that comprise the database.

3.2. Application of Distribution Fitting Approaches to Extratropical Storm Analysis

The application of the distribution fitting methods to extratropical data is essentially a peak-over-threshold (POT) approach akin to the application of POT to a tide gauge record. In the application, all events with surges above the threshold are selected from historical gauge record of T years, yielding N events. A non-parametric distribution fitting method such as EST can be fit to the data using plotting positions, or a parametric distribution can be fit to the data using a regression analysis combined with plotting positions. More directly, fitting techniques such as the Method of Moments or the Maximum Likelihood Method can be used. The overall storm rate N/T is used to scale the frequency of occurrence.

The same distribution fitting approaches can be used on peak water levels from simulated extra-tropical storms. Peak events are identified through review of all tide gauge records within, and adjacent to the study area. The extratropical storms associated with those events are identified and simulated using a suitable wind and circulation model assuming representative wind fields are available for each event. At any location within the study area, the peak elevation recorded for each simulated event constitutes the N-event POT-type dataset that is subsequently fitted with a distribution.

For parametric distribution fitting, the tail of the curve naturally extends the distribution beyond the period of record, providing an estimate of the surge elevation associated with less frequent events. For the EST, the random-walk boot-strapping method can be applied to extend the distribution. In this case both the input and response vectors are the peak surge during the storm event.

Uncertainty can also be included in the analysis. As an example of a parametric distribution fitting, if the simulated storm data were computed using mean tide elevation, a random tide elevation can be selected and added to the peak surge to produce another dataset. The random tide selection can be applied M times to produce M data sets, each of which is fitted with the distribution. The average over the M distribution fit constitutes the mean, or ‘best estimate,’ curve. The variability about the mean, or the standard deviation of the curves, quantifies the uncertainty due to the phasing of the tide and peak surge. This approach can be applied to evaluate the uncertainty associated with any storm parameter or physical process. This approach can be applied to evaluate the uncertainty associated with any storm parameter or physical process.

4. Monte Carlo Method

As discussed above for the JPM approach, the Monte Carlo method is based on probability distributions established for the parameters needed to characterize a storm. Unlike the JPM, however, these probability distributions are not discretized. Instead, storms are constructed by randomly choosing values for each parameter by generating random values uniformly distributed between 0 and 1, and then entering the cumulative distributions at those values and selecting the corresponding parameter values. Each storm selected by this Monte Carlo procedure is simulated with the hydrodynamic model. Simulating a large number of storms in this way is analogous to simulating a long period of history, with the frequency connection established through the rate of storm occurrence estimated from a local storm sample.

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